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Abstract

What is the effect of factor mobility on income convergence? Why are population flows so persistent? Extending the neoclassical growth model to allow for mobile labor, in a long run steady state, individuals and firms receive equal levels of utility and profits across localities. But frictions in the form of a cost to installing capital proportional to the rate of gross investment and an analogous cost to moving proportional to the rate of net migration effect extended equilibrium transition paths during which rents will be associated with living and owning capital in some localities relative to others. The speed of income convergence depends mostly on capital mobility (i.e. the installation cost) and is relatively insensitive to the degree of labor mobility. Persistent population flows result from relatively small changes in local productivity or quality of life, even with very high labor mobility; but even when population is relatively distant from its steady-state level, wages and land prices remain relatively close to their steady-state levels. Local growth theory admits several other results. The speed of income convergence varies considerably in a neigborhood very close to the steady state. Consumption smoothing causes steady-state asset wealth and hence steady-state population density to be history dependent. Steady-state land prices rise at exactly the right rate to offset any flow of population from high productivity to high quality-of-life locales as per capita income rises.

Keywords: Economic Growth, Income Convergence, Factor Mobility, Migration, Spatial Equilibrium, Compensating Differentials.

JEL Codes: O41, R11, F43, J61

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1 Introduction

How do we think about economic growth in a local context? High levels of labor mobility would seem to place an upper bound on the degree to which per capita income levels, let alone per capita income growth rates, can differ among localities. Which suggests a second question, particularly relevant as European nations remove formal barriers to internal labor migration: How important is labor mobility to per capita income convergence? Finally, given that it is one of the most salient characteristics among localities across which there is a high level of labor mobility, Why are population and employment flows so persistent?

To answer these requires an explicitly local theory of economic growth. By "local" I mean to connote a small open economy which exists within a larger integrated macroeconomy characterized by high labor mobility. Thus a locality may correspond to a city or region within a nation state or even, as suggested by the example of the European Union, to a nation state itself. Being small, a locality can take tradable output prices and interest rates as given; conditions within the locality itself, on the other hand, determine local nontradable prices, local wage levels, and local population.

In what follows I extend neoclassical growth theory (Ramsey, 1928; Cass, 1965; Koopmans, 1965) to model such a locality. Doing so requires the introduction of a fixed resource other than population to capture that localities are limited in scope; land is the obvious candidate. That individuals' location decisions may depend on local attributes in ways other than how such attributes affect local wages and land prices requires the introduction of local quality of life into utility functions. In a long run steady state, localities must offer resident firms and individuals the same level of profits and utility available to them elsewhere within the integrated macroeconomy. But frictions, in the form of an installation cost to capital proportional to the rate of gross investment and an analogous moving cost proportional to the rate of net migration effect an extended equilibrium transition path during which rents will be associated with owning installed capital and living in certain localities relative to others. Herein I will focus on the dynamics experienced by a single such locality while assuming that the integrated rest-of-world economy is already at its steady state. The dynamics by which the overall system reaches a steady state thus remains an important question for future research

The choice of the neoclassical framework, in which the level of technology is exogenously determined, is motivated by the smallness assumption on localities and serves as a natural first pass at modeling local growth theory. Increasing returns to scale and local knowledge spillovers, however, suggest that endogenous growth models should prove equally applicable to a local setting (Krugman, 1991, Ades and Glaeser, 1995).

Various elements of a neoclassical local growth theory already exist within the economics literature. In particular, Mueser and Graves (1995) contend that the instantaneous equating of utility and profits across localities assumed by static theories of locational choice is unrealistic; instead, they argue that population and firm locational movements must be proportional to utility and profit differentials. More formally, Braun (1993) introduces labor mobility into the neoclassical growth framework by assuming that labor flows are proportional to the difference in the net present value of labor income.

Addressing the questions raised above, local growth dynamics suggest that the speed of income convergence depends mostly on capital mobility (i.e. the installation cost) and is relatively insensitive to the degree of labor mobility. Persistent population flows result from relatively small changes in local productivity or quality of life, even with very high labor mobility. As to how to think about growth in a local context, flows of population serve as the local observable that best measures changes in underlying productivity and quality of life.

Local growth theory admits several other results. The speed of income convergence varies considerably in a neigborhood very close to the steady state. Consumption smoothing causes steady-state asset wealth and hence steady-state population density to be history dependent. Steady-state land prices rise at exactly the right rate to offset any flow of population from high productivity to high quality-of-life locales as per capita income rises.

The paper proceeds as follows: Section 2 lays out a static model of locational choice. Section 3 develops the dynamic model of local growth incorporating the static model as its long run steady state but introducing frictions which effect an extended equilibrium transition path. Section 4 discusses the transitional dynamics following a negative shock to locality's capital stock. Section 5 discusses the transitional dynamics following changes to local total factor productivity and to local quality of life. Section 6 concludes.

2 Static Theory

Both the static and dynamic models view the world as a system of small, open economies with high cross-border factor mobility. Optimizing individuals within such an integrated economy choose to live in the locality offering the highest utility; optimizing firms choose to locate in the locality offering the highest profits. The resulting shifts in local labor supply and labor demand effect a spatial equilibrium characterized by identical levels of utility and profits across localities. (Rosen, 1979; Roback, 1982; Gyourko and Tracy, 1989, 1991).

More formally, the equating of utility levels across localities can be captured by,

$$U\left(p,w;\text{quality of life}\right)=\left\{\max_{c,n}\ u\left(c,n;\text{quality of life}\right)\text{s.t. }\ c+pn\leq w\right\}=\overline{U} \tag{1}$$

$$\begin{aligned} u_c\left(\cdot\right) &> 0;\ u_{cc}\left(\cdot\right)<0 \\ u_n\left(\cdot\right) &> 0;\ u_{nn}\left(\cdot\right)<0 \\ u_{\text{quality}}\left(\cdot\right) &> 0;\ u_{c,\text{quality}}\left(\cdot\right)=u_{n,\text{quality}}\left(\cdot\right) \end{aligned}$$

Here, $U(\cdot)$ represents an indirect utility function with the price of housing services, p, and the wage level, w, as its arguments, and quality of life as a shift parameter. The underlying (direct) utility function, $u(\cdot)$, is increasing in consumption of a tradable good, c, and nontradable housing services, n. With the tradable good as numeraire and normalizing the per capita quantity of inelastically supplied labor to one, individuals face the budget constraint that their tradable consumption plus their expenditure on housing services not exceed the wage rate. The first two sets of derivative restrictions just establish that utility is strictly increasing and concave with respect to both the tradable and nontradable goods. The third set of derivative restrictions establishes that a higher quality of life indeed raises individual utility but that it does not alter the relative utility trade-off between the tradable and nontradable goods.

The equal profit condition is captured by,

$$\Pi\left(w,\bar{r};\text{productivity}\right)=\left\{\max_{K,L}\ F\left(K,L;\text{productivity}\right)-wL-\left(1+\bar{r}\right)K\right\}=\overline{\Pi} \tag{2}$$

$$F_K\left(\cdot\right)>0;\ F_L\left(\cdot\right)>0;\ F_{\text{productivity}}\left(\cdot\right)>0$$

$\Pi(\cdot)$ represents a firm profit function which, given local wages and an exogenous interest rate, is the maximized value of firm production less its wage and interest bill. The derivative assumptions establish that the marginal products of capital and labor always remain positve and that higher productivity indeed raises output.

Normalizing the quantity of land to one, and assuming a flow of one unit of housing services from each unit of land, a representative locality's resource constraint gives,

$$nL = 1 \tag{3}$$

Note that for the representative locality, L measures both population and population density. Generalizing to localities with different (fixed) quantities of land, L should be interpreted only as population density. For the analysis which follows, the key theoretical results are that

$$\frac{\mathrm{d}w}{\mathrm{d\ productivity}} > 0; \qquad \frac{\mathrm{d}p}{\mathrm{d\ productivity}} > 0; \qquad \frac{\mathrm{d}L}{\mathrm{d\ productivity}} > 0 \tag{4}$$

$$\frac{\mathrm{d}w}{\mathrm{d\ quality\ of\ life}} = 0; \qquad \frac{\mathrm{d}p}{\mathrm{d\ quality\ of\ life}} > 0; \qquad \frac{\mathrm{d}L}{\mathrm{d\ quality\ of\ life}} > 0 \tag{5}$$

Proofs of (4) and (5) are deferred until Appendix A.

An important caveat to the preceding partial derivatives is that several rely on the exclusion of land from the production function, $F(\cdot)$. When land is included in the production function as in Roback (1982) and Gyourko and Tracy (1989, 1991), the derivative of the output-denominated wage with respect to quality of life, $\frac{\mathrm{d}w}{\mathrm{d\ quality\ of\ life}}$, is negative: in order to attain their reservation level of profits, firms pay a lower output-denominated wage as compensation for the higher output denominated land price. With land excluded from the production function, the derivative of the output-denominated wage with respect to quality of life is zero but the derivative of the real, consumption-denominated wage with respect to quality of life is negative. More importantly, the positive derivative of population density with respect to productivity may not follow. Higher productivity causes an outward shift in both firms' and individuals' demand for housing services (due, respectively, to an increase in the marginal product of land and the income effect of higher output denominated wages). Together with the resulting increase in the price of housing services, higher productivity may cause the actual aggregate quantity of housing services purchased by firms and the per capita quantity of housing services purchased by individuals to either increase or decrease. When land is absent from the production function, Appendix A proves that for individuals, the price effect dominates the income effect so that per capita land service consumption drops and hence population must increase. But if firms increase their aggregate use of land, then even a decrease in per capita land service consumption may not be sufficient to prevent a decrease in population. Hence local growth theory as presented herein may not be a very good model for economies in which a land is a large input into tradable production (e.g. economies which are primarily agricultural).

3 Dynamic Theory

The local growth model which follows embeds the static spatial equilibrium above as its long run steady state but assumes frictions in the form of a cost to installing capital proportional to the rate of gross investment and an analogous utility cost to moving proportional to the rate of net migration which effect an extended equilibrium transition path during which rents will be associated with owning installed capital and living in certain localities relative to others.

Though straightforward, the current model is a challenge to present due to the large number of associated variables and equations. Hence I have chosen to highlight just the setup and the results. All derivations are available upon request. The remainder of this section is divided into seven subsections: individual utility functions and behavior, firm production functions and behavior, land price determination, characteristics of the *row* steady state, the decision by individuals to migrate to or from locality i, the implied dynamics of locality i's transition, and — finally — the characteristics of the locality-i steady state including comparative steady-state "statics" associated with changes in exogenous parameters.

3.1 Individuals

I assume a small open economy inhabited by a continuum of individuals with collective mass $L_i(t)$. These individuals need not be identical; but if they are not, I must adopt a structure sufficient to allow for the admittance of a representative local agent. Herein, such structure is indeed present as are assumptions that insure that all locally-residing individuals are identical; per capita variables can thus be interpreted as pertaining either to a representative agent or to all local individuals. A key difference from the standard neoclassical framework is that in addition to the consumption of private output goods, individuals also derive utility from the consumption of private housing services, $n_i(t)$, and non-congestible quality-of-life amenities, $\text{quality}_i(t)$. Lifetime utility is given by,

$$U_i(t) = \int_t^\infty \Big((1-\zeta)\log\left(c_i\left(s\right)\right) + \zeta \log\left(n_i\left(s\right)\right) + \eta \log\left(\text{quality}_i\left(s\right)\right) \mathrm{e}^{-\rho(s-t)} \Big)\, \mathrm{d}s \tag{6}$$

As in the neoclassical model, individuals face an instantaneous asset accumulation constraint. With the output good as numeraire, $p_i(t)$ as the rental price of housing services, and *assuming absentee landlords*, this is given by,

$$\frac{\mathrm{d}}{\mathrm{d}t}\text{assets}_i\left(t\right) = r \cdot \text{assets}_i\left(t\right) + w_i\left(t\right) - c_i\left(t\right) - p_i\left(t\right) n_i\left(t\right) \tag{7}$$

In addition, individuals face the lifetime budget constraint that the net present value of their output and land-service consumption not exceed their current wealth which is itself the sum of their asset wealth and the net present value of their wages.

$$\int_t^\infty \left(c_i\left(s\right) + p_i\left(t\right) n_i\left(s\right)\right) e^{-r(s-t)}\, \mathrm{d}s \leq \text{total\_wealth}_i\left(t\right) \tag{8}$$

$$\text{total\_wealth}_i\left(t\right) \equiv \text{assets}_i(t) + \text{labor\_wealth}_i\left(t\right)$$

$$\text{labor\_wealth}_i\left(t\right) \equiv \int_t^\infty w_i\left(s\right) \mathrm{e}^{-r(s-t)} \mathrm{d}s$$

Setting up and solving for individuals' optimal behavior, at any point in time they will devote the fraction ρ of their total wealth on current consumption; of this, they will spend the fraction $(1-\zeta)$ on the tradable output good and the remaining fraction ζ on housing services. The actual quantity of housing services consumed depends on its rental price, the value of which will be determined endogenously.

$$c_i\left(t\right) = \rho\left(1-\zeta\right) \text{total\_wealth}_i\left(t\right) \tag{9a}$$

$$n_i\left(t\right) = \frac{\rho\zeta \text{total\_wealth}_i\left(t\right)}{p_i\left(t\right)} \tag{9b}$$

The additive separable utility form in (6) along with the optimal output and housing consumption functions, (9a) and (9b), allow for an easy decomposition of individuals' lifetime utility into a function, $f(\,\cdot\,)$, whose arguments are exogenous to locality i, along with elements that depend separably on individuals' wealth, the time path of local land rental prices, and the time path of local quality of life.

$$\begin{aligned} U_i\left(t\right) &= \mathrm{f}\left(\rho,\zeta,r\right) + \frac{\log\left(\text{total\_wealth}_i\left(t\right)\right)}{\rho} - \zeta \int_t^\infty \log\left(p_i\left(s\right)\right) \mathrm{e}^{-\rho(s-t)} \mathrm{d}s \qquad (10a) \\ &\quad + \eta \int_t^\infty \log\left(\text{quality}_i\left(s\right)\right) \mathrm{e}^{-\rho(s-t)} \mathrm{d}s \\ U_i\left(t\right) &= \mathrm{f}\left(\rho,\zeta,r\right) + U_{\text{wealth},i}\left(t\right) + U_{\text{price},i}\left(t\right) + U_{\text{quality},i}\left(t\right) \qquad (10b) \end{aligned}$$

Since the economy-wide adding up constraint that the sum of individuals' asset wealth must equal the aggregate capital stock does not apply to our locality, it becomes necessary to track the evolution of locality-i asset wealth. Assuming for the moment no effect on mean asset wealth from migration into or out of locality i, (7), (9a) and (9b) imply that per capita asset wealth evolves according to,

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{assets}_i(t) = w_i(t) + (r-\rho)\,\mathrm{assets}_i(t) - \rho\cdot\mathrm{labor\_wealth}_i(t) \tag{11}$$

In fact I will make the key assumption that anyone migrating into locality i has the same contemporary asset wealth as the current mean in i which implies that (11) will hold in equilibrium.[1]

3.2 Firms

Within locality i are a number of firms, each with access to a constant-returns-to-scale (CRS) production function. As CRS implies an indeterminate firm size, I write instead the aggregate locality i production function,

$$Y_i(t) = A_i(t)\,K_i(t)^{\alpha}\left(L_i(t)\,\mathrm{e}^{xt}\right)^{1-\alpha} \tag{12}$$

$A_i(t)$ captures locally-applicable total factor productivity while x captures the economy-wide, exogenous rate of labor-augmenting technological progress. As in the static model, housing services are excluded from firms' production function. Firms still "care" about land service prices as these affect the wage firms need to pay to attract workers. Output can be rewritten in intensive form based on the number of "effective" labor units as,[2]

$$\hat{y}_i(t) = A_i(t)\,\widehat{k}_i(t)^{\alpha} \tag{13}$$

A firm's objective is to maximize the net present value of its profits. Along the lines of Abel (1982) and Hayashi (1982), I assume an adjustment cost to installing capital; hence

[1] Asset wealth's main importance is its role in determining local land service prices as shown below. Given a homothetic specification of utility as in (6), in fact what matters for land prices is *mean* local asset wealth. Allowing for individuals whose asset wealth differs, the evolution of mean asset wealth is the same as in (11), only with the addition of a term that captures the difference between current mean asset wealth and the asset wealth of current migrants.

[2] In general, lower case variables are meant to connote the per capita normalization of aggregate levels and "hatted" variables, the normalization by the level of labor-augmenting technological progress, e^{xt}.

the total cost of installed capital is $1+\Phi\left(\frac{I_i(t)}{K_i(t)}\right)$ where $\frac{I_i(t)}{K_i(t)}$ is the rate of gross investment and $\Phi(\cdot)$ is increasing in its argument. In particular, I specialize the adjustment cost as a linear function of the rate of gross investment, $\Phi\left(\frac{I_i(t)}{K_i(t)}\right) = \frac{b_{K,i}}{2}\frac{I_i(t)}{K_i(t)}$. The parameter $b_{K,i}$ captures the magnitude of the capital installation cost. Letting $b_{K,i}$ go to zero captures a world in which capital can be costlessly installed and uninstalled. Firms face the dynamic constraint that the change in their level of capital stock is just the sum of their level of gross investment less any capital depreciation. Locality-i firms' dynamic optimization problem can be written in "current-value" "Jacobian" form as,

$$\begin{aligned} \mathrm{J}_{\mathrm{firms},i}(t) &= A_i(t)\,K_i(t)^{\alpha}\left(L_i(t)\,\mathrm{e}^{xt}\right)^{1-\alpha} - w_i(t)\,L_i(t) \\ &\quad - \left(1+\frac{b_{K,i}}{2}\frac{I_i(t)}{K_i(t)}\right)I_i(t) + q_{K,i}(t)\cdot\left(I_i(t)-\delta K_i(t)\right) \end{aligned} \tag{14}$$

The two choice variables are the firms' level of employment, $L_i(t)$, and their level of gross investment, $I_i(t)$.[3] The co-state variable, $q_{K,i}(t)$, captures the current shadow value of the marginal unit of installed capital. That the production function, (12), is CRS and that the installation cost function, $\Phi(\,\cdot\,)$, depends only on the ratio $\frac{I_i(t)}{K_i(t)}$ together imply that this "marginal" q in fact equals "average" q, the ratio of the value of total installed capital to its uninstalled replacement cost. (Hayashi, 1982). The solution to (14) is standard and so is omitted.

3.3 Land Price Determination

Local housing services are assumed to flow at the fixed aggregate rate, $N_i(t)$. More realistically, housing services might be supposed to be produced using a locality's fixed supply of land combined with various amounts of other intermediate inputs. The inclusion of a housing-service production function is a priority for future research; for the moment the justification for a fixed supply of housing services is that it captures that land-service production above some threshold level is doubtlessly characterized by a very high marginal cost.

With housing-service supply permanently fixed and population instantaneously fixed, mean per capita housing-service consumption, $n_i(t)$, must equal $\frac{N_i}{L_i(t)}$. The current rental

[3] Note that as expressed in (14), these are in fact aggregate locality-i variables. Given the indeterminacy of scale associated with CRS production functions, the distinction is immaterial. Also, given the assumption that the local labor market clears, $L_i(t)$ is in fact predetermined; what is not predetermined is the wage that will make firms willing to employ $L_i(t)$ units of labor.

price of housing services, $p_i(t)$, is just the price which realizes this level of housing-service demand. Using (9b) and the definition of total wealth, the price of housing services which clears the market can be written as,

$$p_i(t) = \frac{\rho\zeta}{N_i} L_i(t) \cdot (\text{assets}_i(t) + \text{labor\_wealth}_i(t)) \tag{15}$$

The sales price of housing services can then be calculated as the net present value of the housing service rental price:

$$\text{value}_{row}(t) \equiv \int_t^\infty p_i(s)\ \mathrm{e}^{-r(s-t)} \mathrm{d}s$$

3.4 ROW Steady State

In contrast to locality i , the remaining rest-of-world economy is assumed to be in its long run steady state. That locality i is small and the rest of the world is large allows for such a dichotomy. The *row* steady state is characterized by standard neoclassical results for a closed economy. Net borrowing among *row* individuals is zero and so mean *row* asset wealth must exactly equal the value of *row* installed capital ($\text{assets}_{row}(t) = q_{row}\ k_{row}(t)$. The interest rate which effects such an equilibrium is given by the sum of individuals rate of time preference and the rate of technological progress ($r = \rho + x$). The equilibrium shadow value of capital, $q_{K,row}$, is exactly that which induces a rate of investment consistent with a constant level of capital per effective worker. This turns out to be,

$$q_{K,row} = 1 + x\ b_{K,row} + \delta\ b_{K,row} \tag{16}$$

The constancy in the steady state of system variables when normalized by the level of technology implies that each of these grows at the exogenous rate of technological progress without such a normalization:

$$\frac{\frac{\mathrm{d}}{\mathrm{d}t} w_{row}(t)}{w_{row}(t)} = \frac{\frac{\mathrm{d}}{\mathrm{d}t}\text{labor\_wealth}_{row}(t)}{\text{labor\_wealth}_{row}(t)} = \frac{\frac{\mathrm{d}}{\mathrm{d}t} k_{row}(t)}{k_{row}(t)} = \frac{\frac{\mathrm{d}}{\mathrm{d}t}\text{assets}_{row}(t)}{\text{assets}_{row}(t)} = \frac{\frac{\mathrm{d}}{\mathrm{d}t} p_{row}(t)}{p_{row}(t)} = x \tag{17}$$

3.5 The Decision to Migrate

In order to keep the rate of net migration between i and *row* to a finite level (and analogous to the installation cost associated with capital investment), I assume a *utility* cost to migrating proportional to the net flows in both the departing and receiving locality.[4] A number of

[4] Similar results could be achieved by assuming an asset wealth cost to net migration which rose at the rate of exogenous technological progress, x. If the asset wealth cost to net migration rose slower than the rate of technological progress, labor mobility would increase with time.

real-world justifications can be offered based on the housing market, local infrastructure requirements, local hostility to large foreign influxes, do-it-yourself moving truck rental prices, local job search, etc. Letting arrows represent the direction of net migration, the utility cost can be formalized as,

$$U^{\text{cost}}_{i\to row} = b_{L,row}\frac{\frac{\partial}{\partial t}L_{row}(t)}{L_{row}(t)} - b_{L,i}\frac{\frac{\mathrm{d}}{\mathrm{d}t}L_i(t)}{L_i(t)} = -b_{L,i}\frac{\frac{\mathrm{d}}{\mathrm{d}t}L_i(t)}{L_i(t)} \tag{18a}$$

$$U^{\text{cost}}_{row\to i} = -b_{L,row}\frac{\frac{\mathrm{d}}{\mathrm{d}t}L_{row}(t)}{L_{row}(t)} + b_{L,i}\frac{\frac{\partial}{\partial t}L_i(t)}{L_i(t)} = b_{L,i}\frac{\frac{\mathrm{d}}{\mathrm{d}t}L_i(t)}{L_i(t)} \tag{18b}$$

The second set of equalities follows from the largeness assumption about *row*.[5] In an equilibrium, the flow between i and *row* must be such that the marginal migrant be indifferent between migrating or not. This will be the case when the utility cost associated with migrating exactly equals the incremental lifetime utility associated with living in the destination location. Defining $\mathrm{d}U_i(t)$ as the utility differential associated with living in i,

$$\mathrm{d}U_i(t) \equiv U_i(t) - U_{row}(t)$$

it follows that the rate of net migration *into* locality i is given by,

$$\frac{\frac{\mathrm{d}}{\mathrm{d}t}L_i(t)}{L_i(t)} = \frac{\mathrm{d}U_i(t)}{b_{L,i}} \tag{19}$$

I still need to define what I mean by a "marginal migrant". While all agents in both i and *row* are assumed to be identical with regards to their inherent characteristics (there are no high-skilled or low-skilled individuals), where they may differ is with regards to their asset wealth; moreover this is a difference that they retain should they choose to migrate. Assume for the moment $\mathrm{d}U_i(t) > 0$ so that there is a positive utility differential associated with living in i. This will induce a net migration flow from *row* to i. The largeness assumption on *row* allows me to avoid making any distinction between marginal and average migrants (i.e. *row* has a sufficient number of residents with any given asset wealth level that all time-t migrants can be assumed to be identical). The question is, what is the level of asset wealth of those *row* individuals who choose to migrate? This is an extremely important issue to which there is no obvious answer.

One possibility is to assume that all *row* individuals have identical asset wealth. But such an assumption implies a certain internal inconsistency: individuals who live in i will

[5] The flow out of or into *row* is just the negative of the flow into or out of i, and L_{row} is an order of magnitude greater than L_i; as long as $\frac{\mathrm{d}}{\mathrm{d}t}L_i(t)$ is of the same order of magnitude as $L_i(t)$, $\frac{\frac{\mathrm{d}}{\mathrm{d}t}L_i(t)}{L_{row}} \approx 0$

in general not have the same asset wealth as those who live in *row* so that for all *row* individuals to have identical asset wealth, it must be the case that no one in the past has ever migrated from i to *row*.

As soon as we allow for heterogenous *row* asset wealth (while maintaining the adding-up requirement that aggregate *row* asset wealth equal the value of aggregate *row* capital stock), it becomes apparent that the utility gains realized by migrating from *row* to i will be greater for some *row* individuals than for others. I will formalize this presently, but the intuition is straightforward. Suppose that land prices and quality of life are identical in both i and *row*, but that the level of wages and labor wealth are higher in i. Since consumption choices are based on total wealth rather than labor wealth, it is those *row* individuals with the lowest asset wealth who would realize the greatest percentage increase in their total wealth by migrating to i. Within the current framework, greater percentage increases in total wealth map to greater level changes in lifetime utility so that poorer individuals (with regards to asset wealth) would be willing to pay a greater utility cost than their richer counterparts to migrate. Conversely, richer individuals would be more willing to suffer a decrease in their labor wealth if i were able to offer a higher level of quality of life. For the formalization, just replace "$\text{assets}_i\,(t)$" in (21a) with "$\text{assets}_{\text{migrant}}\,(t)$" and notice that $\frac{\partial \mathrm{d}U_{\text{wealth},i}}{\partial \text{assets}_{\text{migrant}}} \lessgtr 0$ as $\text{labor\_wealth}_i \gtrless \text{labor\_wealth}_{row}$.

Returning to the question of who migrates, a second possibility is to assume that it is those who have the most to gain. Leaving aside the consequent modeling issue of the need to specify an exogenous distribution function over *row* asset wealth, migration by those who gain the most would imply an inverse correlation between asset wealth and labor wealth, a prediction that would be difficult to reconcile with the real world.

The possibility I choose instead is to assume that migrants from *row* to i have asset wealth equivalent to the contemporary mean in i. Consistent with Tiebout's (1956) hypothesis that migration sorts a heterogenous population into more homogenous sub-populations, migration in the present case sorts individuals according to their asset wealth. Several justifications are possible. The most compelling is probably that in the real world, zoning laws place limits on the quantity of housing services that individuals can buy; having the same asset wealth as current residents, in-migrants desire the same quantity of housing services.[6]

[6] More problematic is reconciling such a zoning explanation with the modeling of in-migrants as raising aggregate demand for housing services thereby causing current residents to decrease their housing-service

In any case, none of the model's results would seem to depend on such Tiebout wealth sorting, and given the homothetic specification of individual utility, any of the alternative assumptions on the asset wealth of actual migrants could be handled in a straightforward manner.

The utility differential associated with living in i relative to *row* can be decomposed using (10b) where each of the right-hand-side terms is defined analogously to $\mathrm{d}U_i(t)$:

$$\mathrm{d}U_i\left(t\right) = \mathrm{d}U_{\text{wealth},i}\left(t\right) + \mathrm{d}U_{\text{price},i}\left(t\right) + \mathrm{d}U_{\text{quality},i}\left(t\right) \tag{20}$$

Using (10a) and the definition of total wealth, these in turn can be written as,

$$\mathrm{d}U_{\text{wealth},i}\left(t\right) = \frac{1}{\rho}\log\left(\frac{\text{labor\_wealth}_i\left(t\right) + \text{assets}_i\left(t\right)}{\text{labor\_wealth}_{row}\left(t\right) + \text{assets}_i\left(t\right)}\right) \tag{21a}$$

$$\mathrm{d}U_{\text{price},i}\left(t\right) = \zeta\int_t^\infty \log\left(\frac{p_{row}\left(s\right)}{p_i\left(s\right)}\right)\mathrm{e}^{-\rho(s-t)}\mathrm{d}s \tag{21b}$$

$$\mathrm{d}U_{\text{quality},i}\left(t\right) = \eta\int_t^\infty \log\left(\frac{\text{quality}_i\left(s\right)}{\text{quality}_{row}\left(s\right)}\right)\mathrm{e}^{-\rho(s-t)}\mathrm{d}s \tag{21c}$$

The quotient in (21a) captures the relative wealth of a potential migrant between i and *row*. As discussed above, migration implies a change only in labor wealth with asset wealth remaining the same. That a potential migrant has asset wealth equal to $\text{assets}_i(t)$ is just the assumption of Tiebout wealth sorting.

Henceforth I will assume that the local quality of life is time invariant. Such an assumption obviously does not allow for congestion effects or for the endogenizing of quality-of-life provision through some public choice mechanism. For the moment, therefore, quality of life should be thought of as exogenously determined — e.g., the weather, or proximity to lakes, the ocean, mountains, etc.

3.6 Dynamics

The dynamic system can now be expressed as a system of seven differential equations in $\{L_i(t),\ \widehat{k}_i(t),\ \widehat{\text{assets}}_i(t),\ q_{K,i}(t),\ dU_{\text{wealth},i}(t),\ dU_{\text{price},i}(t),\ \widehat{\text{value}}_i(t)\}$. The first three of these, $\{L_i(t),\ \widehat{k}_i(t)$, and $\widehat{\text{assets}}_i(t)\}$, — are "state" variables which are instantaneously fixed (i.e. they are can not "jump"). The remaining four, $\{q_{K,i}(t),\ dU_{\text{wealth},i}(t),\ dU_{\text{price},i}(t)$, and $\widehat{\text{value}}_i(t)\}$, are "co-state" variables which can jump, but only in reaction to unexpected system shocks. The dynamic system is mutually recursive with respect to all of the variables

consumption (but not expenditure).

with the exception of $\widehat{\text{value}}_i(t)$; none of the remaining system variables depends on the evolution of $\widehat{\text{value}}_i(t)$ and so it could be dropped from the system without further loss of information; I retain $\widehat{\text{value}}_i(t)$ because it maps to a key local observable. The actual expressions for the differential equations are deferred until Appendix C.

Any remaining endogenous variables can be calculated from the contemporary values of these seven system variables along with the various exogenous parameters.

3.7 Local Steady State and "Comparative Statics"

The local steady state can be derived by setting each of the seven system differential equations just discussed, (C.1a) – (C.1g), equal to zero and solving for the state variables. The actual expressions are again deferred until Appendix C. The steady-state values of two of these, $\{\widehat{k}_i(t), q_{K,i}(t)\}$, are determinate in that they can be expressed as a function of exogenous parameters alone. The remaining five system variables, $\{L_i(t), \widehat{\text{assets}}_i(t), dU_{\text{wealth},i}(t), dU_{\text{price},i}(t), \text{ and } \widehat{\text{value}}_i(t)\}$, collectively have one degree of freedom in the sense that in addition to the exogenous parameters, the steady state value of one of these needs to be known to determine the steady state values of the other four.

The "extra" degree of freedom results from the fact that the overall system is subject to history dependence. The intuition on how this arises is straightforward. Consider two localities, i and j, identical in all exogenous parameters, but having a different history of local development. In particular, at some point in the distant past (say at $t = 0$) i experienced a "helicopter drop" of installed physical capital. (Never mind how capital could be installed from a helicopter, or indeed whether helicopters even existed at $t = 0$.) At this same point in the distant past, j experienced an "artillery drop" which destroyed a large portion of its installed capital base. (Thankfully, no one was injured.) From (C.2b), it follows that the steady-state levels of labor income will be identical between the two localities. But during the transitions to their respective steady states, i's residents have high current relative to permanent income whereas j's residents have low current relative to permanent income. Consumption smoothing leads i's residents to accumulate but j's residents to decumulate asset wealth during the transition to the steady state. It immediately follows that in these steady states, i's residents have a higher asset wealth than j's residents. In the present scenario, it turns out that the steady-state price of housing services will be identical between the two localities. (If the steady-state land service price differed, there would be an incentive to migrate between the two.) The higher asset wealth of i's residents, however, means that they will be purchasing a higher steady-state *quantity* of housing services. With equal aggregate endowments of land, this can only be if the population of i is smaller than that of j.

The illustrative story above should bring home the point that the one degree of freedom with respect to the system variables in no way implies that there is any possibility of "choice" over locality steady states (other than altering the exogenous parameters). On the contrary the system is fully determined; it is just that this determination is based both on the "static" exogenous parameters as well as the nature of the equilibrium transition to the steady state.

Comparative steady state "statics" can now be calculated for various local observables. Table 1 contains a summary. Both steady-state population and land value are increasing with respect to local productivity and local quality of life and decreasing with respect to the local capital installation cost. Output-denominated wages and capital intensity are increasing with respect to productivity, neutral with respect to quality of life, and decreasing with respect to the local capital installation cost. Note that the comparative statics for the history-dependent observables, population and land value, assume a constant steady-state level of local asset wealth. When total factor productivity is equal across localities as in the helicopter versus artillery drop example above, increases in steady-state asset wealth are associated with higher land values and lower population. More generally, an increase in local steady-state asset wealth is associated with an increase in both land values and population when local productivity is low relative to *row* and a decrease in both land values and population when local productivity is high relative to *row*. Appendix C includes a more detailed discussion.

That the steady state of the present system is characterized by the constancy of each of the system variables is by no means automatic. Rather, defining the steady state of a system as the constancy of the *time derivatives* of its component variables, then a slight relaxing of the dynamic model's assumptions would lead to a steady state in which there was migration. The intuition is again straightforward. With positive technological progress ($x > 0$), consumption is rising and hence the marginal utility of consumption is falling with time. One implication is that individuals should be increasingly willing to trade off lower labor wealth for a higher level of quality of life thereby inducing a steady-state population flow from high TFP localities to high quality-of-life localities. Herein this does

Table 1

Steady-State Response of Endogenous
Variables to Variation in Exogenous Parameters

	L_i^*, $\widehat{\text{value}}_i^*$	$\widehat{k}_i^*$, $\widehat{w}_i^*$	$q_{K,i}^*$
A_i	$+$	$+$	0
quality_i	$+$	0	0
$b_{K,i}$	$-$	$-$	$+$
$b_{L,i}$	0	0	0

not occur both because housing services are assumed to flow at a fixed rate and because the elasticity of intertemporal substitution is the same for housing services and local quality of life; as a result, the steady-state price of housing services rises at exactly the right rate to offset the increasing marginal utility of local quality-of-life consumption relative to output consumption. The utility value of local quality of life is thus fully capitalized into steady-state land-service prices. Suppose, however, that the quantity supplied of housing services responded to changes in the price of housing services or that the elasticity of intertemporal substitution were greater for land-services than for quality of life. In either case the steady-state rate of land-service price increases would be insufficient to offset increasing demand for local quality-of-life consumption. Rather, a steady-state utility gradient would exist between low and high quality-of-life locales as would steady-state migration from the former to the latter. Note that I am presuming the existence of a steady state, a result which awaits the extension of the model to explicitly include such alternative assumptions.

4 Factor Mobility and Income Convergence

What is the effect of factor mobility on income convergence? Numerical solutions to the local growth system detailed above richly characterize the time paths of income and other local observables following a negative shock to a locality's capital stock. The speed at which income converges back towards its steady-state level turns out to depend mostly on the degree of capital mobility (i.e. the capital installation friction) and is relatively insensitive to the degree of labor mobility. This qualitative result is robust across a wide range of calibrations.

A "negative capital shock" is meant to connote any set of circumstances which leaves a locality with a low installed capital base relative to an unchanged steady-state level. Literally interpreted, negative capital shocks might correspond to natural and man-made disasters. More metaphorically, negative capital shocks might correspond to changes in technology or the terms of trade which disproportionately affect the installed capital base of some localities relative to others but which do not fundamentally alter relative productivity or quality of life: for instance, changes in manufacturing techniques in steel production on certain areas in the Midwest United States. Similarly, the effect of communist government on the eastern states of Germany from 1945 to 1989 might correspond to an extended negative capital shock.

Negative capital shocks are not meant to connote asynchronous business cycles. While the dynamics following a negative capital shock are suggestive, the effect of factor mobility on asynchronous cyclical fluctuations awaits the extension of the present model to explicitly include unemployment.

4.1 Representative Time Path

Figure 1 sketches the time paths of population, wages, and land prices following a shock to a locality's capital stock that leaves its initial wage at 60 percent of its steady-state level. Immediately following the shock (at time 0), population begins to rapidly flow out of the locality (Panel A). The decrease in local wages causes both the sales and rental price of land to discretly jump downward at the time of the capital shock; following the shock the rental price of land continues to fall driven by the outflow of population (Panel C). Not shown is the large inflow of gross capital stock that the negative capital shock induces (due to an increase in the the marginal revenue product of capital and hence its shadow value). The population outflow and the capital inflow both tend to cause the marginal product of labor and hence wages to increase; higher wages along with lower land prices eventually reverse the population outflow. As population flows back into the locality, gross capital formation remains sufficiently positive to allow wages to continue to converge back towards their steady-state level. The sales and rental price of housing services also gradually return to their steady-state level.

As laid out in the theory section above, the local growth system is characterized by history dependence. Following a negative capital shock, consumption smoothing causes individuals to decumulate assets; along with the assumption of Tiebout wealth sorting, this decumulation implies that locality per capita asset wealth will fall in turn lowering per capita demand for housing services. Steady-state equilibrium in the land market therefore requires a higher population in the locality than was present prior to the income shock. In the calibration shown in Figure 1, immediately following a negative capital shock, population flows out of a locality at a 3.5 percent annual rate. From an initial level of exactly 1, population reaches a nadir of 0.858 after 12 years. Population returns to its initial level approximately 54 years after the initial shock and continues to rise eventually reaching a level of 1.058 in its new steady state. Not shown is the decumulation of asset wealth which in the new steady state is 0.447 of its pre-shock level (normalized for the level of labor-augmenting technology); the rate of asset decumulation is directly proportional to the gap between current and permanent income so that it is most rapid immediately following the negative capital shock.

The rate at which wages converge to their pre-shock level is somewhat rapid. Immediately following the shock, the population outflow combined with the gross capital inflow combine to cause wages to grow at an 8.3 percent annual rate.[7] In less than 6 years they have returned to 80 percent of their steady-state level; in just over 12 years they have returned to 90 percent of their steady-state level; and in approximately 40 years they have returned to 99 percent of their steady-state level. Note that at this 40-year benchmark, population and land prices remain substantially below their pre-shock levels (which for population applies *a fortiori* to its level relative to its new steady state).

An alternative way of measuring the rate at which wages return to their steady state is to formally define a "speed of convergence" as,

$$\Lambda\left(\log y\left(t\right)-\log w^{*}\right)\equiv\frac{-\frac{\mathrm{d}}{\mathrm{d}t}\left(\log w\left(t\right)-\log w^{*}\right)}{\log w\left(t\right)-\log w^{*}}$$

That is, Λ is the rate at which wages close the log gap to their steady state. By this metric, the speed of convergence steadily falls from $\Lambda = 0.163$ when wages are at 60 percent of their steady-state level to $\Lambda = 0.097$ when wages are at 95 percent of their steady-state level. As the relative wage level approaches the steady state, the rate at which the speed of convergence is falling off increases. At the actual steady state itself (by which I mean within an ϵ neighborhood of it), the speed of convergence has fallen to just $\Lambda = 0.040$.

[7] Note that here and below, wages are measured relative to their *row* level which implicitly normalizes them by the level of labor augmenting technology.

While a decreasing speed of convergence commonly characterizes neoclassical growth systems, in general the rate at which the speed falls off is decreasing as income approaches a steady state (i.e. the speed has a negative first but positive second derivative). In the present case, a negative second derivative of convergence speed as income increases implies that the speed of convergence can vary considerably even in a neigborhood quite close to the steady state. For calibrations with lower labor mobility than in Figure 1, the speed of convergence drops from rates near $\Lambda = 0.09$ when wages are at 99 percent of their steady-state level to less than $\Lambda = 0.01$ at the steady state. Indeed, measuring the speed of convergence only at the steady state, the local growth system appears to achieve a calibration that has proved elusive for the neoclassical growth model: a narrow capital share along with a low steady-state shadow value of capital and a slow speed of convergence; but examining the speed of income convergence at income levels more than negligably below the steady state, the slow speed of convergence disappears.

Given the way it varies, Λ does not prove a very useful measure of convergence speed. Instead I will use wage levels at various benchmark times and the time to reach various benchmark wage levels as my main comparison metrics for the speed of income convergence.

4.2 Base Calibration

The representative time path sketched in Figure 1 is based on a single calibration of a model imbibing a large number of exogenous parameters. In exploring the qualitative robustness of the time paths with respect to variations in the levels of these exogenous parameters, a natural starting point is to try to choose "base" parameter values that allow the model to approximately match real world observables.

In general I have used parameter values which are the same as in Barro and Sala-i-Martin (1995). As enumerated on the right hand side of Figure 1, these include the capital depreciation rate, $\delta = 0.05$, the rate of time preference, $\rho = 0.02$, and the rate of exogenous technological progress, $x = 0.02$. Barro and Sala-i-Martin include two parameterizations of the capital share, α. A first narrow capital share parameterization, $\alpha = 0.30$, corresponds to a literal interpretation of physical capital and approximately matches the share of national income accounted for by rental income, profits, and interest payments. The main problem with a narrow capital share parameterization is that the speed of convergence, as defined by Λ above, tends both to be "too high" and to decline rapidly as income approaches its steady-state level. A partial solution to both problems comes from assuming a broad capital share, for instance $\alpha = 0.75$, corresponding to a more metaphorical interpretation of capital to include human capital. The solution is only "partial", first because a broad concept of capital implies investment rates somewhat higher than we actually observe and, second, because a decreasing speed of convergence remains in the open-economy version of the neoclassical model (King and Rebello, 1993; Rappaport, 1998).

For the purposes of local growth theory, a broad capital share is especially problematic. To the extent that broad capital is interpreted as human capital, the appropriate friction would be the labor rather than the capital friction: that is that a qualitatively important difference between physical and human capital is the mobility of the latter. Even so, it is possible to argue for a broader share of *fixed* capital in output production than is implied by the national income accounts. One possibility is that a factor-income derived capital share fails to account for tax-financed public-sector capital (to the extent that it was debt-financed, the associated interest payments would contribute to the implied capital share). Another possibility is that human capital is in part locality-specific so that its adjustment may be more appropriately modeled by the capital rather than the labor friction. In the alternative calibration section below, I will consider the case of a moderately broad capital share, $\alpha = 0.60$.

Unique to the local growth model is a parameter capturing the share of consumption expenditure devoted to housing services. A 20 percent housing share, $\zeta = 0.20$, roughly matches the correponding figure from the U.S. national income accounts. As housing services proxies more generally for local nontradable goods (and to the extent that local land prices may contribute to the final price facing consumers for locally sold tradable goods), this housing share is probably conservative. The main effect of raising the housing share parameter is to dampen the response to the capital shock. Consider the limit as the housing share approaches 1: in this case there is no response to the negative capital shock. While individuals' output denominated wage falls, their real wage as deflated by the price of housing services remains unchanged (i.e. the rental price of housing services falls to exactly the level which allows individuals to continue to consume the same amount of housing services); as individuals derive no utility from output consumption, their utility remains unchanged and hence there is no incentive for outmigration.

A second parameter unique to the local growth model is the relative utility individuals derive from local quality of life. For the numerical solutions involving capital shocks, quality of life is assumed to be equal across localities and so the choice of η does not matter.

Given the motivating question, how does factor mobility affect income convergence, the parameters which are inherently of the most interest are those that govern the capital installation and net migration frictions — $b_{K,i}$, $b_{K,row}$, and $b_{L,i}$. As a starting point, I map these friction parameters to more intuitive measures of mobility. In particular, for given rates of depreciation and exogenous technological progress, the capital frictions map one-to-one to the steady-state shadow values of capital $q^*_{K,i}$ and $q^*_{K,row}$. Henceforth I will assume that this shadow value is equal across localities; the case where the steady-state shadow value of capital varies across localities is qualitatively similar to the case of variations in productivity across localities explored below. Similarly, for a given rate of time preference, ρ, the labor friction maps one-to-one to to the relative wealth necessary to induce a 0.01 rate of net migration, ω. (So $\omega = \exp(0.01 \cdot \rho \cdot b_{L,i})$.)

From Blanchard, Rhee, and Summers (1993), we believe that the shadow value of capital tends to remain relatively close to one. However low steady-state shadow values of capital near one tend to be associated with implausibly high rates of income convergence. For instance, in the closed economy analog to the local growth model (i.e. with no labor mobility and with local savings financing local investment), a steady-state shadow value of capital $q^*_K = 1.14$ implies a steady-state speed of income convergence $\Lambda = 0.091$. In the open-economy version of the model (i.e. with no labor mobility but with the ability to borrow and lend at an exogenous interest rate), it implies a steady-state speed of income convergence $\Lambda = 0.175$. While the closed-economy convergence rate is consistent with some recent panel-data estimates (Islam, 1995, Caselli, LeFort, and Esquival, 1995), the open-economy convergence rate is not. Moreover both the open-economy and closed-economy convergence rates rise with distance from the steady state (i.e. the speed of convergence is falling into the steady state as discussed above). To allow for slower income convergence, I choose for a base calibration, a higher steady-state shadow value of capital, $q^*_K = 1.56$. Even so, the speed of convergence as measured at the steady state remains somewhat high ($\Lambda = 0.067$ and $\Lambda = 0.093$ for the closed- and open-economy versions of the neoclassical model, respectively). Hence the alternative calibrations section below also considers the case of a considerably higher steady-state shadow value of capital, $q^*_K = 3.24$ (which implies closed- and open-economy steady-state speeds of convergence $\Lambda = 0.048$ and $\Lambda = 0.059$).[8]

For the labor mobility parameter, no good empirical estimates exist to use as a benchmark. To be sure, numerous researchers have looked at the empirical relationship between wage levels and population flows. Barro and Sala-i-Martin (1991), for instance, find a moderate positive relationship between initial wage levels and subsequent population growth; but Blanchard and Katz (1995) and Rappaport (1999) find no consistent relationship between wage levels and population flows. The failure to find such a relationship should not be surprising given that local growth theory suggests the relative real wealth levels rather than relative wages are what should drive population flows, and even then, only after controlling for locality-specific quality of life. That is, local growth theory suggests a data generating process of the form,

$$\text{net\_migration}_{i,t} = \beta \log\left(\text{real\_total\_wealth}_{i,t}\right) + \mathbf{X}_{i,t}\mathbf{\Gamma} + \varepsilon_{i,t} \tag{22}$$

where $\mathbf{X}_{i,t}$ captures locality-specific attributes that contribute to quality of life, and real total wealth is a function of the future time paths of output-denominated wages and land prices along with current asset wealth,

$$\text{real\_total\_wealth}_{i,t} = \mathrm{f}\left(\{w_{i,s}\}_{s=t}^{\infty}, \{p_{i,s}\}_{s=t}^{\infty}, \text{asset\_wealth}_{i,t}\right)$$

Output-denominated wages exaggerate differences in relative real wealth in at least three different ways. First is that individuals' consumption bundles include local nontradables (e.g. housing services) whose price level should be proportional to local wages. Second is the convergence over time of wage levels as documented in Barro and Sala-i-Martin (1991). Third is that relative real wealth includes not just labor wealth but also asset wealth. The need to control for locality-specific quality of life follows because differences in relative real wealth may represent nothing more than compensation for differences in quality of life. Controls for differences in local productivity are not included as productivity affects population flows only via real wealth differences; including such productivity controls would therefore seem to bias the coefficient on real total wealth towards zero. Of course in practice, many of the local attributes which affect local quality of life, $\mathbf{X}_{i,t}$, will also affect local productivity.

[8]The speeds of convergence just listed are based on a narrow capital share, $\alpha = 0.30$, and the remaining parameter values enumerated in Figure 1 along with a unitary elasticity of intertemporal substitution which matches the implicit assumption in the local growth model.

In the absence of good estimates of the coefficient β from the data generating process, (22), above, I am forced to rely largely on introspection in calibrating labor mobility. As a base calibration, I set the level of labor mobility such that $\omega = 1.01$: that is, that a 1 percent real wealth differential is sufficient to induce a 1 percent rate of net migration. Such a calibration would be implied by an estimate of β close to 1. Using output-denominated wage rather than real wealth, and with only a sparse set of controls, $\mathbf{X}_{i,t}$, Barro and Sala-i-Martin estimate a version of (22) for each decade, 1860 through 1990. Their highest estimate of β is 0.0439 which corresponds to $\omega = 1.25$. For all the reasons just discussed, such an estimate is likely to greatly understate the degree of labor mobility. In the subsection which follows, I explore the effects of varying ω to capture levels of labor mobility both much higher and much lower than the base calibration.

4.3 Alternative Calibrations: The Affect of Factor Mobility on Income Convergence

Finally, then, to directly address the question, how does factor mobility affect income convergence? Figure 2 shows the time paths of population, gross capital, wages, and land prices following a negative capital shock under alternative "high" and "low" labor mobility calibrations. In the high labor mobility regime, $\omega = 1.00125$, just a 1/8 percent difference in real wealth will induce a 1 percent rate of net migration; under the low labor-mobility regime, $\omega = 1.08$, an 8 percent difference is needed to do the same. In terms of the labor friction parameter, $b_{L,i}$, the high regime has 64 times the mobility of the low regime. All other parameters are the same as in Figure 1.

Unsurprisingly, the rate of population outflow immediately following the capital shock is much greater under the high labor mobility regime than under the low one with initial rates of outmigration of 15.9 percent versus just 0.6 percent, respectively (Figure 2 Panel A). With high labor mobility, population eventually drops to a minimum 68 percent of its initial level 7 years after the shock before beginning to flow back. With low labor mobility, the population outflow continues for a longer period but is much shallower; a minimum population 97 percent of its initial level is reached 16 years after the shock. With high labor mobility, the return population flow is also much more rapid and by year 46, population under the high regime comes to exceed that under the low regime. But greater asset deccumulation under the low mobility regime implies that in the final steady state, it will have the higher population. (This second intersection of the population loci is not shown as it takes place more than 100 years after the initial shock.)

In terms of the speed at which income converges back to its long run steady state, the high initial outflow of labor in the high mobility regime quickly lowers local labor supply and drives up the marginal revenue product of labor. At the same time, however, the large labor outflow drives down the marginal revenue product of capital and thus serves as a disincentive to gross capital formation. As a result, gross capital formation is lower under high labor mobility than it is under low labor mobility; immediately following the negative capital shock, the rates of gross capital formation are 20.4 percent and 25.9 percent, respectively (Figure 2 Panel B). Measured relative to its initial level, 10 years after the negative capital shock, extensive capital stock is 10 percentage points lower with high labor mobility than with low labor mobility (i.e. the vertical gap between the loci); after 20 years the difference is 12 percentage points. From a post-shock level 18 percent of its pre-shock level, it takes 15 years for extensive capital to reach 80 percent of its pre-shock level with high labor mobility but just 10 years to do so with low labor mobility.

The inverse correlation of labor and capital flows following capital shocks implies a substitutability between labor and capital mobility. For a given level of capital mobility, decreases in labor mobility and hence population flows are associated with increases in gross capital formation (Table 2 column 4 moving down across rows within a given panel); for a given level of labor mobility, decreases in capital mobility and hence gross capital formation are associated with increases in population flows (Table 2 column 3 moving down across panels for a given row).

Of more inherent interest than extensive capital is the convergence of wages from their pre-shock low back to their steady state. The higher initial outflow of population with high labor mobility contributes to a positive effect of labor mobility on income convergence; the higher growth rate of extensive capital with low labor mobility contributes to a negative effect of labor mobility on income convergence. Figure 2 Panel C shows the combined effect. Immediately following the capital shock, wages grow faster with high labor mobility than with low; the initial growth rates are 0.109 versus 0.080, respectively. After 10 years, wages have returned to 89.9 percent of their pre-shock level with high labor mobility versus 86.5 percent of their pre-shock level with low labor mobility (the vertical gap between loci). For wages to reach 90 percent of their pre-shock level takes 10.1 years with high labor mobility versus 13.0 years with low labor mobility (the horizontal gap between loci). Table 2 Panel B summarizes speed of convergence measures for alternative levels of labor mobility under the base calibration capital share of production, $\alpha = 0.30$, and shadow value of capital, $q_K^* = 1.56$. The remaining panels show the effect of varying labor mobility on the speed of income convergence under alternative calibrations of the shadow value of capital and capital share of production. Note that within a panel, each row represents a doubling of the labor mobility friction relative to the row above; the final row shows the speed of convergence measured in a corresponding neoclassical, open-economy model in which population is assumed fixed.

What is surprising is the extent to which the time paths of wages under high versus low mobility remain quite close to each other. So for the base calibration in Table 2 Panel B, moving from the highest row, where just a 1/16 percentage point wealth difference induces a 1 percent rate of net migration, to the next-to-last row, where a 32 percentage point wealth difference is needed to do the same, the relative wage 10 years after the negative capital shock drops from 0.903 to 0.863; the time until wages reattain 90 percent of their steady-state level increases from 9.6 years to 13.2 years. Such changes would seem qualitatively small when compared with the more than one-thousand-fold increase in the labor mobility friction. Moving to the last row in which labor mobility is completely absent causes virtually no further change in the speed of income convergence measures.

Measured to a point closer to the steady state, the speed of income convergence actually slows with increasing labor mobility. For instance, under the base capital mobility and capital share calibration, twenty years after the capital shock relative wages are 0.955 with the “base” labor mobility but only 0.953 with “high” labor mobility. Similarly, the time it takes for wages to reattain 99 percent of their steady-state level continually falls from 47.4 years under the “high” labor mobility calibration to 40.7 years under the “base” calibration to 37.3 years under the “low” labor mobility calibration. (Table 2 Panel B, Columns 11 and 15)

Figure 3 Panel A plots the relative wage 10 years after the negative shock against decreasing levels of labor mobility. The horizontal axis is denominated such that each horizontal unit corresponds to a doubling of the labor mobility friction. The three loci correspond to “high”, “base”, and “low” levels of capital mobility with steady-state shadow values of capital equal to 1.14, 1.56, and 3.24 respectively (corresponding to Table 2, Panels

A through C). That the loci are negatively sloped captures that the speed of convergence as measured by relative wages 10 years after the shock is indeed increasing with labor mobility. That the loci are relatively flat captures that the effect of labor mobility on income convergence is relatively small. Measured to points closer to the steady state (for instance, t = 40), the loci are actually positively sloped. Figure 3 Panel C mirrors Panel B by measuring the speed of convergence by the time it takes for wages to reattain 90 percent of their steady-state level. Here, that the loci are positively sloped captures the increase in convergence speed with labor mobility; that they are relatively flat captures the relative insensitivity of income convergence to labor mobility. Measured to points closer to the steady state (for instance, a relative wage 99 percent of its steady-state level), the loci are negatively sloped.

In contrast to labor mobility, the degree of capital mobility exerts a powerful influence on the speed of income convergence. Figure 3 Panel B plots the relative wage 10 years after the negative capital shock against decreasing levels of capital mobility. The horizontal axis is scaled such that each horizontal unit corresponds to a doubling of the installation cost friction. (All panels of Figure 3 are scaled to be visually comparable.) Except at very high levels of capital mobility, the wage level is steeply decreasing as the capital installation cost increases. For the base level of labor mobility ($\omega = 1.01$), a doubling of the capital installation cost such that the steady-state shadow value of capital increases from $q_K^* = 1.14$ to $q_K^* = 1.28$ is associated with a decrease in the 10-year relative wage level from 0.946 to 0. 911. Figure 3 Panel D mirrors Panel B by plotting the time it takes for wages to reattain 90 percent of their steady-state level. For the base level of labor mobility, the same doubling of the capital installation cost causes the time to do so to increase from 6.8 to 9.2 years. And unlike the case of increases in labor mobility, increases in capital mobility always contribute to a faster speed of convergence, regardless of where this is measured.

An alternative illustration of the relationship between income convergence and factor mobility is captured by the contrast in the vertical gaps between loci in Figure 3 Panels A versus B. In Panel A, each locus represents a quadrupling of the capital friction relative to the locus above it; the vertical gaps represent the difference in relative wages 10 years after the negative capital shock for various levels of labor mobility. In Panel B, each locus represents an eight-fold increase of the labor friction relative to the locus above it; here the vertical gaps represent the difference in relative wages ten years after the shock for various levels of capital mobility. That the vertical gaps are much greater in Panel A than in Panel B (even though the difference in relative frictions between adjacent loci is twice as geat in Panel B) captures that the speed of income convergence is much more sensitive to variations in capital mobility than it is to variations in labor mobility. The same conclusion follows from the contrast in the vertical gaps between loci in Figure 3 Panels C versus D.

The insensitivity of the speed of income convergence to the degree of labor mobility is a moderately robust qualitative result. The lower the level of capital mobility, the more sensitive is the speed of income convergence to variations in labor mobility. In Figure 3 Panels A and C, this can be seen in the steeper slopes of the high capital friction ($q_K^* = 3.24$) loci; in Figure 3 Panels B and D, it can be seen in the increase in vertical gaps between adjacent loci as capital mobility decreases. Increases in the capital share of production, α, also make income convergence more sensitive to variations in labor mobility. With a combination of a broad capital share, $\alpha = 0.60$, and a high capital friction, $q_K^* = 3.24$, the relative wage 10 years after a negative capital shock falls from 0.855 with "high" labor mobility to 0.746 with "low" labor mobility (Table 2 Panel F). However, as one of the main motivations for using a broad capital share calibration is to avoid using a high capital friction calibration, this latter example may not be relevant. With the combination of a broad capital share and a low capital friction, $q_K^* = 1.14$, the relative wage level 10 years after the shock is 0.928 with "high" labor mobility falling only to 0.890 with "low" labor mobility; 20 years after the shock relative wages are slightly lower with high labor mobility than with low (Table 2 Panel D). [9]

In general, labor mobility proves a weak substitute for capital mobility but capital mobility proves a powerful substitute for labor mobility. The horizontal gaps between loci in Figure 3 Panels B and D capture the increase in capital mobility needed to offset the eight-fold decrease in labor mobility between adjacent loci. Moving from right to left in Panel B illustrates that to maintain the same speed of income convergence measured by relative wages 10 years following the capital shock requires only a very small increase in capital mobility to offset a decrease in labor mobility from $\omega = 1.01$ to $\omega = 1.08$. For the

[9] Just as income convergence is more sensitive to labor mobility the lower the level of capital mobility, so too is income convergence more sensitive to capital mobility the lower the level of labor mobility. In Figure 3 Panels B and D, this can be seen in the steeper slopes of the low labor mobility ($\omega = 1.08$) loci; in Figure 3 Panels A and C, it can be seen in the increase in the vertical gaps between adjacent loci as labor mobility decreases.

four-fold decrease in capital mobility between adjacent loci in Panels A and C, in general there is no increase in labor mobility sufficient to maintain a constant speed of income convergence however measured.

From a welfare perspective, residents' utility immediately following a negative capital shock is strictly increasing in the level of labor mobility. Comparing the high versus low labor mobility regimes, higher relative wages during the early part of the transition path dominate lower relative wages during the latter part of the transition. In addition, the greater outflow of population with high labor mobility causes a greater decrease in the price of local housing services which also lowers the utility loss from the negative capital shock. Of course to the extent that local housing is owned by local residents (as opposed to the model's assumption of absentee landlords) higher levels of labor mobility would be associated with greater losses in local asset wealth. Whether such an "asset wealth" effect of labor mobility is sufficient to cause welfare to become decreasing in the level of labor mobility awaits the extension of local growth theory to explicitly incorporate local land ownership.

5 The Response to Changes in Productivity and Quality of Life

In addition to capital shocks, the two other main sorts of shocks that localities are likely to experience are changes in productivity and changes in quality of life. In particular, I have in mind that localities are characterized by an inherent set of characteristics, e.g. their weather, their location relative to the sea and navigable waterways, their topography, etc. While the attributes themselves are constant across time, it seems reasonable to suppose that the attributes' contributions to productivity and quality of life may vary with time. In the 19th-century, navigable rivers served as a major source of commercial transportation within the continental United States; over the course of a 20th-century, railways and later trucking have almost completely replaced river-borne commerce. And so in the 19th-century, location near a navigable river bestowed a considerable productivity advantage to local producers of tradable goods; at some point during the 20th-century, this productivity advantage eroded. Similarly, in the 19th-century, high desert temperatures made the U.S. Southwest a relatively inhospitable place to live. But the invention of air conditioning in the early 20th-century and its widespread adoption following World War II have led many to consider the Southwest to have a high quality of life.

For an integrated economy in its long run steady state, the static spatial equilibrium outlined in Section 2 allows for wage and land price differentials to be used to infer the contributions of various local attributes to productivity and quality of life (Rosen, 1979; Roback, 1982; Gyourko and Tracey, 1989, 1991). An obvious question, then, is how reasonable is an identifying assumption that integrated economy is in its long run steady state? For the United States, in fact, there is strong evidence against such an assumption. One of the most salient facts of local U.S. growth is the persistence of population flows. Blanchard and Katz (1992), Barro and Sala-i-Martin (1991), and Borts (1960) all document such persistence for various population measures and various time periods for the U.S. states. Glaeser, Scheinkman, and Shleifer (1995) document the persistence of U.S. city population growth over the period 1950 to 1990; and Rappaport (1999) shows the same for U.S. counties over the period 1930 to 1990. While persistent population flows are not prima facie evidence of deviation from a long run steady state,[10] such deviation is the simplest explanation for the persistent population flows. Greenwood et al. (1991) explicitly use net population flows and local wage levels to conclude that the system of U.S. localities is away from its long run steady state. The convergence of per capita income across the U.S. states documented by Barro and Sala-i-Martin (1991) reinforces such a conclusion.

Numerical solutions of the dynamics following local productivity and quality-of-life shocks highlight two key results: First is that even very small frictions to labor and capital mobility imply that productivity and quality-of-life shocks will induce population flows which persist over a long period. Second is that although a locality may be "far away" from it steady state when measured by population, wage levels and land sales prices may nevertheless be quite close to their steady-state levels. Hence using variations in wages and housing prices to make inferences on underlying productivity and quality of life seems a reasonable procedure even for a system on an equilibrium transition path rather than in a long run steady state.

[10] For instance, the case of per capita income growth inducing a steady-state flow from high productivity to high quality-of-life localities as discussed in the theory section above.

5.1 Representative Time Paths

Figures 4 and 5 show the time paths of local observables following positive shocks to local total factor productivity and local quality of life. Both figures share the base calibration discussed in the capital shock section above: a steady-state shadow value of capital equal to 1.56 and a moving cost requiring a one percent real wealth differential to effect a one percent rate of net migration along with a narrow capital production share ($q_K^* = 1.56$, $\omega = 1.01$, $\alpha = 0.03$). Results from alternative calibrations, summarized in Tables 3 and 4, are discussed in the subsection immediately below.

The positive total factor productivity shock illustrated in Figure 4 is of a size that will result in a 5 percent increase in steady-state wages. Because capital intensity is increasing in TFP, the actual size of the positive productivity shock is smaller: 3.5 percent given the 30 percent capital share of production (see C.2b). As capital intensity is instantaneously fixed, the jump in relative wages accompanying the productivity shock is therefore from exactly 1 to 1.035 (Figure 4 Panel B). Along the transition path, wages rise the remaining 1.5 percentage points; their growth rate, however, remains quite small: from just 0.06 percent immediately following the shock asymptoting back to zero (Figure 4 Panel D). Similarly, most of the increase in housing sales prices occurs concurrent with the increase in TFP. The relative housing sales price jumps from exactly 1 to 1.176 concurrent with the shock thereafter rising to 1.248 along the transition path (Figure 4 Panel C); the growth rate of housing sales price falls from 0.23 percent immediately following the shock back to zero along the transition path (Figure 4 Panel D).

Unlike wages and housing prices, population cannot "jump"; as a result the growth rate of population remains much higher along the transition path to the new steady state. Immediately following the shock, population flows in at a 0.73 percent annual rate dropping to 0.49 percent after 10 years; 20, 30, and 40 years after the shock, the rate of net migration remains at 0.33, 0.22 and 0.15 percent, respectively. These magnitudes are relatively large for the response to a one-time 3.5 percent increase in TFP considering that the standard deviation in the average annual rate of net migration across U.S. counties over the period 1970 to 1990 is 1.4 percent.

The positive quality-of-life shock illustrated in Figure 5 is of a size such that individuals are willing to pay a 20 percent premium for housing services while still attaining their reservation level of utility; this size was chosen to approximately match the increase in population accompanying the positive productivity shock sketched in Figure 4. Note that with quality-of-life changes defined by the size of the premium individuals are willing to pay for housing services, the actual choice for the parameter weighting the utility individuals derive from quality of life, η, is immaterial: the higher is η , the lower the increase in quality of life needed. The absolute size of the quality-of-life shock also depends on the housing consumption share, ζ; the higher is ζ, the larger the increase in quality of life needed. Thus for a given absolute increase in quality of life, the larger the housing share of consumption, the smaller the increase in land prices and population (see (C.2g) and (C.2a)).

While a 20 percent increase in land prices may sound large, consider that across the 50 largest U.S. cities in 1990, the median rent for a two-bedroom apartment ranges from \$373 in Cleveland to \$877 in San Francisco. To the extent that such price variations reflect underlying differences in quality of life, an increase in quality of life for which individuals would be willing to pay a 20 percent premium represents approximately one-seventh of the quality of life difference between San Francisco and Cleveland. Of course, observed price variations also reflect productivity differences as well as heterogeneous workforce and housing stock characteristics between places. And so alternatively, one can introspect willingness to pay for various quality-of-life attributes such as good weather, nice views, oceanfront location, etc.

The increase in quality of life induces a persistent inflow of population: immediately following the shock population flows into the locality at a 0.89 percent rate dropping to 0.52 percent after 10 years; 20, 30, and 40 years after the shock, the rate of net migration remains at 0.32, 0.21, and 0.14 percent, respectively (Figure 5 Panel D).

The population inflow drives down wages. Immediately following the shock these decrease at a 0.11 percent rate. Wages reach a minimum 99.4 percent of their *row* level approximately 13.3 years after the quality-of-life shock; thereafter they gradually return to their steady-state level of unity (Figure 5 Panel B). Note that during this latter period both population and wages are increasing. A static labor supply versus labor demand framework suggests an inference of an increase in labor demand from such a combination; but the quality of life increase underlying the present dynamics seems more akin to an increase in labor supply.

The population inflow also increases demand for housing services. While the rental price of housing services rises gradually,[11] housing sales price, which is just the net present value of housing rental prices, jumps concurrent with the quality-of-life shock: from exactly 1 to 1.135. Along the transition path, the housing sales price gradually rises the remainder of the way to 1.20 (Figure 5 Panel C); its growth rate falls from 0.23 percent immediately following the shock back to zero along the transition path.

5.2 Alternative Calibrations: Factor Mobility Following Productivity and Quality-of-Life Shocks

A first main point from the time series following the positive productivity and quality-of-life shocks sketched in Figures 4 and 5 is the extended time frame of the transition paths. As shown in the base calibration labor mobility row of Panel B in Tables 3 and 4, the time required for population to close 50 percent of the distance to its steady state is 17.1 and 15.2 years for the productivity and quality-of-life shocks, respectively; to close 75 percent of the distance to the respective steady states takes 34.4 and 31.8 years. In both cases the speed of convergence of population towards its steady state — that is, the rate at which it closes the log gap — remains relatively constant along the transition path. Numerical solutions bear out that for identical calibrations, the speed of convergence of population is insensitive to the size of the initial productivity or quality-of-life shock. So even for much smaller changes in productivity and in quality of life, persistent population flows will result.[12]

Tables 3 and 4 show that the high persistence of population flows following productivity

[11] In fact, housing rental price actually drops very slightly concurrent with the shock. Its rise thereafter is due entirely to the inflow of population. But for the fixed population residing in the locality when the positive quality-of-life shock occurs, the future inflow of population causes labor wealth to drop and hence so does their demand for housing services.

[12] The relative constancy of the speed of convergence *along the transition path* tends to hold except for very high levels of labor mobility. With high labor mobility the speed of convergence tends to fall along the transition path; for instance with the base level capital friction, $q_K^* = 1.56$ and high labor mobility, $\omega = 1.00125$, the speed of population convergence falls from $\Lambda = 0.131$ to $\Lambda = 0.055$ along the quality-of-life transition path. The relative constancy of the speed of convergence *across different-sized initial shocks* tends to hold even with high labor mobility. Note that the constancy of the speed of population convergence along the transition path stands in sharp contrast to the behavior of the speed of income convergence following capital shocks discussed in the section above The speed of population convergence following capital shocks varies even more than does income convergence given that population initially flows away from its steady-state level.

and quality-of-life shocks is an extremely robust qualitative result. In general, higher levels of either the capital installation friction or the labor mobility friction tend to decrease the speed at which population converges towards its steady state. For a given level of labor mobility, decreases in capital mobility always slow the rate of population convergence (moving down across panels for a given row in Tables 3 and 4); similarly, for a given level of capital mobility, decreases in labor mobility always slow the rate of population convergence (moving down across rows within a given panel in Tables 3 and 4). This complementarity between capital and labor mobility for population convergence contrasts with their substitutability for income convergence following capital shocks discussed in the previous section.

Given this complementarity, examining population convergence with very low levels of both frictions places a lower bound on the persistence of population flows: with $q_K^* = 1.14$ and $\omega = 1.0025$, the time to close 75 percent of the initial distance of population from its steady state following the productivity and quality-of-life shocks is 16.3 and 14.9 years respectively (Tables 3 and 4, Panel A). So even with extremely high levels of both capital and labor mobility, relatively small changes in underlying total factor productivity or quality of life induce quite persistent flows of population.

That wages remain relatively close to their steady state following productivity shocks is a moderately robust result. Regardless of the level of factor mobility, wages jump concurrent with the productivity increase by exactly the amount of this increase. The remaining gap of wages from their steady-state level is due to the increase in steady-state capital intensity brought about by the increase in productivity. The percentage of the steady-state increase in relative wages which accompanies the initial increase in productivity is decreasing in the capital share of production: $initial\_relative\_wage = final\_relative\_wage^{(1-\alpha)}$. For the narrow capital share calibration, $\alpha = 0.30$, 69 percent of the final increase in relative wages occurs concurrent with the productivity shock; but for the broad capital share calibration, $\alpha = 0.60$, only 39 percent of the final increase in relative wages accompanies the shock.

While capital and labor mobility do not affect the initial jump of wages concurrent with a productivity shock, they do affect the rate at which wages close the remaining gap to their steady-state level. For a given level of labor mobility, decreases in capital mobility always decrease the rate at which wages converge to their steady-state level (Table 3, moving down across panels for a given row). For a given level of capital mobility, decreases in labor mobility increase the rate at which wages converge to their steady state (Table 3, moving down across rows within a given panel). The latter result can be decomposed into the slower inflows of population accompanying low labor mobility dominating the slower inflows of gross capital stock that such slower population flows beget.

Following quality-of-life shocks, wages remain very close to their steady state across all tested calibrations. The largest deviation of wages from their steady state occurs under the combination of a broad capital share, $\alpha = 0.60$, low capital mobility, $q_K^* = 3.24$, and very high labor mobility, $\omega = 1.000625$; here wages reach a minimum 97 percent of their steady state level 7.2 years following the quality-of-life shock (Table 4 Panel F). For the base calibration, wages drop to just 99.4 percent of their steady-state level (Table 4 Panel B).

That housing sales prices remain relatively close to their steady state following productivity and quality-of-life shocks is somewhat fragile. This is surprising given that housing sales price is an inherently forward-looking metric (i.e. that it is a discounted present value). The explanation is that the main determinant of housing rental prices is current population which, as just outlined, is slow to adjust to changes in productivity and quality of life. The lower the levels of capital and labor mobility, the slower the rate of population adjustment and hence the farther will be initial post-shock housing sales prices from its steady-state level. For the base calibration, housing sales price jumps 71 percent and 68 percent of the distance to its new steady state concurrent with the productivity and quality-of-life shocks, respectively. With the same level of capital mobility, $q_K^* = 1.56$, but with low labor mobility, $\omega = 1.08$, these drop to 50 percent and 38 percent; holding labor mobility constant, going from the base level to low capital mobility, $q_K^* = 3.24$, the quality-of-life relative jump drops to 64 percent. Similarly moving from a narrow to a broad capital share also causes the portion of the distance to its new steady state by which initial land sales price jumps to become smaller.

Summing up, despite extended equilibrium transition paths following productivity and quality of life shocks during which population may remain far away from its steady state, an assumption that wages and housing sales prices are close to their steady-state levels does not seem unreasonable. The numerical results show that for housing sales prices, a necessary condition for such an assumption is that there be sufficient levels of capital and labor mobility. But the underlying assumption of a fixed supply of housing services suggests that the numerical results be interpreted as an upper bound on the degree to which housing prices can deviate from their steady-state level. Under more realistic supply conditions, housing prices are likely to remain significantly closer to their steady-state level along the transition path.

An important caveat is that following capital shocks, both wages and land sales price may deviate substantially from their long run steady-state levels; moreover, this deviation can persist for a long time. Hence it behooves researchers using a compensating differential framework to consider whether the local attributes being correlated with wages and house sales prices may in fact be correlated with local "capital shocks". So, for instance, cyclical downturns in the prices of various agricultural commodities might lead to a false inference that weather conditions conducive to the growth of such commodities contribute negatively to productivity and to quality of life. In general allowing for this type of capital shock interpretation is much easier handled (and often dismissed) than allowing for the possibility that local wages and land prices do not fully incorporate the local attribute contributions to productivity and quality of life.

Complementing the compensating differential framework, population flows can identify shifts in local productivity and local quality of life. The partial correlations of net migration with various local attributes should pickup the *change* in contributions from such attributes to local productivity and local quality of life. The persistence of population flows following such changes suggests that such an exercise should be relatively insensitive to the exact time period over which net migration is examined.

6 Conclusions

In order to think about growth in a local context, the neoclassical growth model is extended to allow for labor mobility. Housing services are included in individuals' utility functions as a source of long run congestion; quality of life is included in individuals' utility functions to capture that individuals may prefer to live in some localities rather than others even after controlling for wages and land prices. Frictions in the form of a cost to installing capital proportional to the rate of gross investment and an analogous moving cost proportional to the rate of net migration effect in extended equilibrium transition path during which rents are associated with owning capital and living in some localities relative to others.

Two main results emerge: first is that the speed of income convergence following capital shocks is relatively insensitive to the degree of labor mobility; outflows of population following such shocks create a disincentive for gross capital formation. Second is that discrete changes in local productivity and in local quality of life induce very persistent flows of population even when the changes are small and even when labor is highly mobile; at the same time wages and house sales prices remain relatively close to their steady-state levels.

Local growth theory admits several other results. The speed of income convergence varies considerably in a neigborhood very close to the steady state. Consumption smoothing causes steady-state asset wealth and hence steady-state population density to be history dependent. With inelastically supplied housing services and assuming equal rates of the elasticity of intertemporal substitution for housing services and quality of life, steady-state land prices rise at exactly the right rate to offset any flow of population from high productivity to high quality-of-life locales as per capita income rises. And even with complete labor mobility (i.e. frictionless labor), away from a long run steady state only utility levels rather than utility flows will be equal across localities.

These various qualitative results suggest a wide-ranging research agenda. At a most basic level is the desirability of exploring their robustness to alternative functional forms. More specifically, because the capital installation cost and labor moving cost serve as the main mechanisms shaping transitional dynamics, improving our understanding of them, both empirically and theoretically, stands out as a major priority. For the labor moving cost, endogenizing housing service provision offers one possible means for doing so. The substitutability of capital mobility and labor mobility following capital shocks and the resulting insensitivity of the speed of income convergence to the degree of labor mobility raises the question, how important is labor mobility in dampening asynchronous cyclical shocks? Addressing this requires the extension of the local growth model to capture cyclical phenomena such as unemployment.

More generally, local growth theory suggests that labor flows are the best observable local analog to per capita income growth across nation states. For nation states, per capita income growth derives either from rising productivity or from capital accumulation. At the local level, both rising productivity and capital accumulation are accompanied by inflows of labor. Such inflows have the additional benefit that they pickup rises in local quality of life. As the ultimate goal of "growth" is the long run attainment of high utility, or so I would argue, there is no good *a priori* reason to exclude quality-of-life considerations from growth theory. While the difficulty of quantifying quality of life largely proscribes its explicit consideration in growth models at the level of the nation state, as soon as one endogenizes locational choice, to exclude quality of life is to assume away an important, nonorthogonal force. As local growth theory illustrates, rising levels of local per capita income may derive from capital accumulation and from rising productivity but also from falling quality of life. And so even after controlling for capital shocks, interpreting the partial correlates of local per capita income growth is especially problematic. In contrast, while population flows can not distinguish between productivity and quality-of-life changes, they do unambiguously identify whether such changes were in a "good" or "bad" direction.

Appendices

A Partial Derivatives with Respect to Productivity and Quality of Life

To establish that $\frac{\mathrm{d}w}{\mathrm{d\ productivity}} > 0$, recognize that $\Pi(\cdot)$ is a profit function. Hence its derivatives with respect to input prices are negative: $\Pi_w(\cdot) < 0$ and $\Pi_{\bar{r}}(\cdot) < 0$. Using the envelope theorem, we know that

$$\begin{aligned} \frac{\mathrm{d}\Pi(\cdot)}{\mathrm{d\ productivity}} &= \frac{\partial \Pi(\cdot)}{\partial\ \mathrm{productivity}} + \Pi_w(\cdot)\frac{\mathrm{d}w}{\mathrm{d\ productivity}} = 0 \\ &= F_{\mathrm{productivity}}(\cdot) + \Pi_w(\cdot)\frac{\mathrm{d}w}{\mathrm{d\ productivity}} = 0 \end{aligned} \tag{A.1}$$

By assumption $F_{\mathrm{productivity}}(\cdot) > 0$. Hence $\frac{\mathrm{d}w}{\mathrm{d\ productivity}} > 0$.

To establish that $\frac{\mathrm{d}p}{\mathrm{d\ productivity}} > 0$, recognize that $U(\cdot)$ is an indirect utility function. Hence its derivative with respect to its resource constraint will be positive, $U_w(\cdot) > 0$, and its derivative with respect to the prices of utility arguments will be negative, $U_p(\cdot) < 0$. Taking the total derivative of $U(\cdot)$, setting this equal to zero, and rearranging gives,

$$\frac{\mathrm{d}p}{\mathrm{d\ productivity}} = -\frac{U_w()}{U_p()}\frac{\mathrm{d}w}{\mathrm{d\ productivity}} > 0 \tag{A.2}$$

To establish that $\frac{\mathrm{d}w}{\mathrm{d\ quality\ of\ life}} = 0$, totally differentiate $\Pi(\cdot)$ and rearrange.

To establish $\frac{\mathrm{d}p}{\mathrm{d\ quality\ of\ life}} > 0$, the envelope theorem gives,

$$\begin{aligned} \frac{\mathrm{d}U(\cdot)}{\mathrm{d\ quality}} &= \frac{\partial U(\cdot)}{\partial\ \mathrm{quality}} + U_p(\cdot)\frac{\mathrm{d}p}{\mathrm{d\ quality}} = 0 \\ &= u_{\mathrm{quality}}(\cdot) + U_p(\cdot)\frac{\mathrm{d}p}{\mathrm{d\ quality}} = 0 \end{aligned} \tag{A.3}$$

By assumption $u_{\mathrm{quality}}(\cdot) > 0$. Hence $\frac{\mathrm{d}p}{\mathrm{d\ quality\ of\ life}} > 0$.

Finally, to show that population density rises with increases in productivity and quality of life, $\frac{\mathrm{d}L}{\mathrm{d\ productivity}} > 0$ and $\frac{\mathrm{d}L}{\mathrm{d\ quality\ of\ life}} > 0$. By the economy resource constraint, (3), this is equivalent to showing that per capita land consumption drops with such changes, $\frac{\mathrm{d}n}{\mathrm{d\ productivity}} > 0$ and $\frac{\mathrm{d}n}{\mathrm{d\ quality\ of\ life}} > 0$.

$$\frac{\mathrm{d}U(\cdot)}{\mathrm{d\ productivity}} = u_c(\cdot)\frac{\mathrm{d}c}{\mathrm{d\ productivity}} + u_n(\cdot)\frac{\mathrm{d}n}{\mathrm{d\ productivity}} = 0 \tag{A.4}$$

$$\frac{\mathrm{d}U(\cdot)}{\mathrm{d\ quality}} = u_{\mathrm{quality}}(\cdot) + u_c(\cdot)\frac{\mathrm{d}c}{\mathrm{d\ quality}} + u_n(\cdot)\frac{\mathrm{d}n}{\mathrm{d\ quality}} = 0 \tag{A.5}$$

Individual utility maximization gives,

$$p = \frac{u_n(\cdot)}{u_c(\cdot)} \tag{A.6}$$

Suppose that $\frac{\mathrm{d}n}{\mathrm{d\ productivity}} > 0$. By (A.4) it follows that $\frac{\mathrm{d}c}{\mathrm{d\ productivity}} < 0$. $u(\cdot)$ is such that $\frac{u_n(\cdot)}{u_c(\cdot)} = p$ must fall. But this violates that $\frac{\mathrm{d}p}{\mathrm{d\ productivity}} > 0$. Hence $\frac{\mathrm{d}n}{\mathrm{d\ productivity}} < 0$. The same argument using (A.5) establishes that $\frac{\mathrm{d}n}{\mathrm{d\ quality\ of\ life}} < 0$.

B Local Growth with Frictionless Labor

An assumption of frictionless labor (i.e. $b_L = 0$) assures that utility *levels* will always be equal across localities. But frictionless labor does not imply that utility *flows* will be identical across localities. On the contrary, in general only in a long-run steady state will utility flows be equal across localities. The key to the proof which follows is that equal utility flows across localities imply equal asset accumulation across localities but that together these two conditions over determine the dynamic system. Rather than equating utility flows instantaneously, frictionless labor implies an intertemporal tradeoff in the sense that living in some localities is associated with higher utility flows today while living in others is associated with higher asset accumulation today (and so higher utility flows in the future).

The instantaneous equating of utility levels across localities implies,

$$\mathrm{d}U_{\mathrm{wealth},i,t} + \mathrm{d}U_{\mathrm{price},i,t} + \mathrm{d}U_{\mathrm{quality},i,t} = 0 \tag{B.1}$$

Using (21a) – (21c) to substitute, the time path of population must satisfy,

$$\{L_{i,s}\}_{s=t}^{\infty} \text{ s.t. } \begin{array}{ll} & \frac{1}{\rho}\log\left(\frac{\int_t^\infty w_{i,s}(K_{i,s},L_{i,s})\mathrm{e}^{-r(s-t)}\mathrm{d}s+\mathrm{assets}_{i,t}}{\int_t^\infty w_{row}(s)\mathrm{e}^{-r(s-t)}\mathrm{d}s+\mathrm{assets}_{i,t}}\right) \\ + & \zeta\int_t^\infty \log\left(\frac{p_{row,s}}{p_{i,s}\left(\{K_{i,v}\}_{v=s}^\infty,\{L_{i,v}\}_{v=s}^\infty\right)}\right)\mathrm{e}^{-\rho(s-t)}\mathrm{d}s \\ + & \eta\int_t^\infty \log\left(\frac{\mathrm{quality}_{i,s}}{\mathrm{quality}_{row,s}}\right)\mathrm{e}^{-\rho(s-t)}\mathrm{d}s \qquad = 0 \end{array} \tag{B.2}$$

Note that (B.2) is the condition for the instantaneos equating of utility levels acoss localities at a single point in time only. Suppose that utility flows are always equal across localities so that (B.2) holds for all t. It follows that asset accumulation must be equal as well (i.e. $\mathrm{assets}_i(t) = \mathrm{assets}_{row}(t)\,\forall t$); if not, individuals could increase their utility by living in a locality with higher asset accumulation for a time after which they switch to a different locality in which their increased asset wealth allows them to finance a higher flow of output and land consumption. Recall that the budget constraint for individual asset accumulation is given by,

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{assets}_{i,t} = r \cdot \mathrm{assets}_{i,t} + w_{i,t} - c_{i,t} - p_{i,t} n_{i,t} \tag{B.3}$$

The first order conditions, (9a) and (9b), imply,

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{assets}_{i,t} = (r-\rho) \cdot \mathrm{assets}_{i,t} + w_{i,t}\left(K_{i,t}, L_{i,t}\right) - \rho \cdot \int_t^\infty w_{i,s}\left(K_{i,s}, L_{i,s}\right) \mathrm{e}^{-r(s-t)} \mathrm{d}s \tag{B.4}$$

In *row*, asset wealth grows at the rate of exogenous technological progress, x (which equals $r-\rho$). Hence the second two terms of the right hand side of (B.4) together must always sum to zero. Hence the future path of population in locality i must satisfy,

$$\begin{aligned} \{L_{i,s}\}_{s=t}^{\infty} \text{ s.t.} \qquad & w_{i,t}\left(K_{i,t}, L_{i,t}\right) \\ - \quad & \rho \cdot \int_t^\infty w_{i,s}\left(K_{i,s}, L_{i,s}\right) \mathrm{e}^{-r(s-t)} \mathrm{d}s = 0 \end{aligned} \tag{B.5}$$

Assume that there exists a time path of population which satisfies (B.5). In general this will not be the same as a time path of population which satisfies (B.2). The system is over determined. Should by coincidence both (B.2) and (B.5) hold simultaneously, a slight perturbation to the parameter ζ will leave (B.5) unaffected but cause (B.2) to no longer hold.

Infinitely-lived agents residing in a low-utility-flow locale need not ever actually realize the future higher utility flows their asset accumulation will allow them. Particularly with a constant elasticity of intertemporal substitution, it is possible that such low-utility-flow residents may be willing to indefinitely postpone their gratification. More generally, life cycle considerations and a decreasing marginal utility of (future) output consumption should eventually cause high-asset-wealth individuals to be less willing to delay gratification than low-asset-wealth individuals. If so, there would need to be a continual reshuffling of agents between high-utility-flow and low-utility-flow locales. With frictionless labor, such a reshuffling is costless. (As long as net migration were zero, such a reshuffling would also be costless in the model in the main text above.)

In the real world, one can find numerous examples of "localities" which allow for a tradeoff of low current utility flows for high future utility flows via high current asset accumulation (e.g. off-shore oil rigs, commercial fishing boats, investment banks, ...)

C Local Growth Equations of Motion and Steady-State Levels

The seven system equations of motion are given by,

$$\frac{\mathrm{d}}{\mathrm{d}t} L_i = \frac{dU_{\mathrm{wealth},i} + dU_{\mathrm{price},i} + dU_{\mathrm{quality},i}}{b_{L,i}} L_i \tag{C.1a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \hat{k}_i = \left(\frac{q_{k,i}-1}{b_{k,i}} - \delta - x - \frac{dU_{\mathrm{wealth},i} + dU_{\mathrm{price},i} + dU_{\mathrm{quality},i}}{b_{L,i}}\right) \hat{k}_i \tag{C.1b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\mathrm{assets}}_i = (1-\alpha)\, A_i \hat{k}_i^a + \rho\, \widehat{\mathrm{assets}}_i - \rho\, \left(\widehat{\mathrm{labor\_wealth}}_{row} + \widehat{\mathrm{assets}}_i\right) \mathrm{e}^{\rho\, dU_{\mathrm{wealth},i}} \tag{C.1c}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} q_i = (\delta+\rho+x)\, q_{K,i} - \alpha A_i \hat{k}_i^{-(1-\alpha)} - \frac{(q_{K,i}-1)^2}{2b_{k,i}} \tag{C.1d}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} dU_{\mathrm{wealth},i} = \mathrm{e}^{\rho dU_{\mathrm{wealth},i}} - \frac{(1-\alpha)\, A_i \hat{k}_i^a + \rho \widehat{\mathrm{assets}}_i}{\rho \left(\widehat{\mathrm{labor\_wealth}}_{row} + \widehat{\mathrm{assets}}_i\right)} \tag{C.1e}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} dU_{\mathrm{price},i} = \zeta \log \left(\frac{\zeta\rho \left(\widehat{\mathrm{labor\_wealth}}_{row} + \widehat{\mathrm{assets}}_i\right) L_i}{\hat{p}_{row} N_i} \right) \tag{C.1f}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\mathrm{value}}_i = \rho\, \widehat{\mathrm{value}}_i - \frac{\zeta\rho \left(\widehat{\mathrm{labor\_wealth}}_{row} + \widehat{\mathrm{assets}}_i\right) L_i\, \mathrm{e}^{\rho dU_{\mathrm{wealth},i}}}{N_i} \tag{C.1g}$$

Setting each of the system equations equal to zero implies steady-state levels,

$$\begin{aligned} L_i^* \quad = \quad & \left(\frac{(1-\alpha)\,(A_i)^{\frac{1}{1-\alpha}} \left(\frac{2\alpha}{\widetilde{b}_{K,i}}\right)^{\frac{\alpha}{1-\alpha}} + \rho \cdot \widehat{\mathrm{assets}}_i^*}{(1-\alpha)\,(A_{row})^{\frac{1}{1-\alpha}} \left(\frac{2\alpha}{\widetilde{b}_{K,row}}\right)^{\frac{\alpha}{1-\alpha}} + \rho \cdot \widehat{\mathrm{assets}}_i^*} \right)^{\frac{1}{\zeta}} \cdot \left(\frac{\mathrm{quality}_i}{\mathrm{quality}_{row}}\right)^{\frac{\eta}{\zeta}} \cdot \\ & \left(\frac{(1-\alpha)\,(A_i)^{\frac{1}{1-\alpha}} \left(\frac{2\alpha}{\widetilde{b}_{K,i}}\right)^{\frac{\alpha}{1-\alpha}} + \rho \cdot \widehat{\mathrm{assets}}_{row}^*}{(1-\alpha)\,(A_{row})^{\frac{1}{1-\alpha}} \left(\frac{2\alpha}{\widetilde{b}_{K,row}}\right)^{\frac{\alpha}{1-\alpha}} + \rho \cdot \widehat{\mathrm{assets}}_i^*} \right) \cdot \left(\frac{N_i}{n_{row}}\right) \end{aligned} \tag{C.2a}$$

$$\widehat{k}_i^* = (A_i)^{\frac{1}{1-\alpha}} \left(\frac{2\ \alpha}{\widetilde{b}_{K,i}}\right)^{\frac{1}{1-\alpha}} \tag{C.2b}$$

where,

$$\widetilde{b}_{K,i} \equiv 2\,(x+\rho+\delta) + (x^2+\delta^2+2x\delta+2x\rho+2\delta\rho) \cdot b_{K,i}$$

$$\widehat{\mathrm{assets}}_i^* = \widehat{\mathrm{assets}}_i^* \tag{C.2c}$$

$$q_{K,i}^* = 1 + (x+\delta)\, b_{K,i} \tag{C.2d}$$

$$dU_{\mathrm{wealth},i}^* = \frac{1}{\rho} \log \left(\frac{(1-\alpha)\,(A_i)^{\frac{1}{1-\alpha}} \left(\frac{2\alpha}{\widetilde{b}_{K,i}}\right)^{\frac{\alpha}{1-\alpha}} + \rho \cdot \widehat{\mathrm{assets}}_i^*}{(1-\alpha)\,(A_{row})^{\frac{1}{1-\alpha}} \left(\frac{2\alpha}{\widetilde{b}_{K,row}}\right)^{\frac{\alpha}{1-\alpha}} + \rho \cdot \widehat{\mathrm{assets}}_i^*} \right) \tag{C.2e}$$

$$dU^*_{\text{price},i} = -\frac{1}{\rho}\log\left(\frac{(1-\alpha)\,(A_i)^{\frac{1}{1-\alpha}}\left(\frac{2\alpha}{\tilde{b}_{K,i}}\right)^{\frac{\alpha}{1-\alpha}} + \rho\cdot\widehat{\text{assets}}^*_i}{(1-\alpha)\,(A_{row})^{\frac{1}{1-\alpha}}\left(\frac{2\alpha}{\tilde{b}_{K,row}}\right)^{\frac{\alpha}{1-\alpha}} + \rho\cdot\widehat{\text{assets}}^*_i}\right) - \frac{\eta}{\rho}\log\left(\frac{\text{quality}_i}{\text{quality}_{row}}\right) \quad \text{(C.2f)}$$

$$\begin{aligned}\widehat{\text{value}}^*_i \;=\; & \left((1-\alpha)\,(A_{row})^{\frac{1}{1-\alpha}}\left(\frac{2\alpha}{\tilde{b}_{K,row}}\right)^{\frac{\alpha}{1-\alpha}} + \rho\cdot\widehat{\text{assets}}^*_{row}\right)\cdot\left(\frac{1}{\rho\, n_{row}}\right)\cdot \\ & \left(\frac{(1-\alpha)\,(A_i)^{\frac{1}{1-\alpha}}\left(\frac{2\alpha}{\tilde{b}_{K,i}}\right)^{\frac{\alpha}{1-\alpha}} + \rho\cdot\widehat{\text{assets}}^*_i}{(1-\alpha)\,(A_{row})^{\frac{1}{1-\alpha}}\left(\frac{2\alpha}{\tilde{b}_{K,row}}\right)^{\frac{\alpha}{1-\alpha}} + \rho\cdot\widehat{\text{assets}}^*_i}\right)^{\frac{1}{\zeta}}\cdot\left(\frac{\text{quality}_i}{\text{quality}_{row}}\right)^{\frac{\eta}{\zeta}}\end{aligned} \quad \text{(C.2g)}$$

Changes in local steady-state asset wealth affect the remaining history-dependent steady-state levels according to,

$$\frac{\mathrm{d}\,L^*_i}{\mathrm{d}\,\widehat{\text{assets}}^*_i} = \begin{matrix}+\\-\end{matrix} \text{ as} \frac{(1-\alpha)\,(A_i)^{\frac{1}{1-\alpha}}\left(\frac{2\alpha}{\tilde{b}_{K,i}}\right)^{\frac{\alpha}{1-\alpha}} + \rho\cdot\widehat{\text{assets}}^*_i}{(1-\alpha)\,(A_{row})^{\frac{1}{1-\alpha}}\left(\frac{2\alpha}{\tilde{b}_{K,row}}\right)^{\frac{\alpha}{1-\alpha}} + \rho\cdot\widehat{\text{assets}}^*_i} \begin{matrix}<\\>\end{matrix} 1-\zeta \quad \text{(C.3)}$$

$$\begin{aligned}\frac{\mathrm{d}\,\widehat{\text{value}}^*_i}{\mathrm{d}\,\widehat{\text{assets}}^*_i} &= \begin{matrix}+\\-\end{matrix} \text{ as} \frac{(1-\alpha)\,(A_i)^{\frac{1}{1-\alpha}}\left(\frac{2\alpha}{\tilde{b}_{K,i}}\right)^{\frac{\alpha}{1-\alpha}} + \rho\cdot\widehat{\text{assets}}^*_i}{(1-\alpha)\,(A_{row})^{\frac{1}{1-\alpha}}\left(\frac{2\alpha}{\tilde{b}_{K,row}}\right)^{\frac{\alpha}{1-\alpha}} + \rho\cdot\widehat{\text{assets}}^*_i} \begin{matrix}<\\>\end{matrix} 1 \\ &= \begin{matrix}+\\-\end{matrix} \text{ as } \frac{A_i}{\tilde{b}_{K,i}} \begin{matrix}<\\>\end{matrix} \frac{A_{row}}{\tilde{b}_{K,row}}\end{aligned} \quad \text{(C.4)}$$

Focusing first on the land price equation, (C.4), a partial effect of an increase in asset wealth is to cause agents to increase their spending on housing services thereby increasing land prices. But the total effect includes in addition that changes in asset wealth affect steady-state population, (i.e. (C.3)). When total factor productivity normalized by the installation cost of capital (i.e. $\frac{A_o}{\tilde{b}_K}$) is sufficiently low in locality i relative to *row*, increases in asset wealth lead to a higher steady-state population for locality i. Hence, both the "partial" and "population" effects of the increase in asset wealth are to raise locality i land prices. On the other hand, when normalized total factor productivity is sufficiently high in locality i relative to *row*, the "partial" and "population" effects are in the opposite direction and so increases in asset wealth can lower steady-state locality i land prices. That the change in sign of the total derivative occurs at a lower value of normalized productivity for population, (C.3), than for land prices, (C.4), naturally follows: when the total derivative for population is exactly zero, the partial effect on land prices of an increase in asset wealth remains positive.

Focusing now on the population equation, consider the case where normalized total factor productivity is lower in i than in *row*. Recall that the assumption of Tiebout wealth sorting implies that $assets_i^*$ is the steady-state asset wealth of *both* i's residents and of potential migrants to i from *row*. The higher the asset wealth of potential migrants, the lower the utility loss from i's low productivity. And so the partial effect of an increase in asset wealth is to make i less unattractive and hence to increase i's population. This partial effect is greater the lower is i's productivity relative to that in *row*. Acting in an opposite direction is the partial effect of higher asset wealth on land prices: for a given population, higher asset wealth implies greater demand for housing services and hence higher land prices in turn making i more unattractive to potential migrants. As long as the left-hand side of the inequality in (C.3) is less than $1-\zeta$, the partial effect dominates this latter "price" effect so that increases in asset wealth increase steady-state population. When the left-hand side of the inequality in (C.3) lies on the interval $[1-\zeta, 1]$, productivity is i still lower than in *row*, but now the price effect dominates the partial effect so that increases in asset wealth decrease steady-state population.

Finally, consider the case where normalized total factor productivity is higher in i than in *row*. Here, the partial effect of an increase in asset wealth on population is negative: the higher the asset wealth of potential migrants, the lower the utility gain from i's high productivity. The negative partial effect of higher asset wealth on steady-state population via land prices remains. Together, these two partial effects imply that the total effect of an increase in asset wealth is to cause steady-state population to decrease. Of course, the latter "partial" effect of higher asset wealth causing an increase in land prices is the opposite of the total effect of asset wealth on land prices as discussed immediately above; but this "reversal" is exactly because the total effect of asset wealth on steady-state population is negative.

A final note: given that asset wealth as it affects steady-state population and land prices in (C.2a) and (C.2a) always enters through a quotient term in which it appears in *both* numerator and denominator, the magnitudes of the total derivatives of steady-state population and land prices with respect to asset wealth tend to be small.

D Convergence with More than One State Variable: Some Algebra

Two key characteristics of the dynamic model are, first, that its steady-state is not uniquely determined but rather depends on history; and second, that the speed of convergence – the rate at which income and population approach their steady-state levels normalized by their distances from their respective steady-state levels – varies greatly, even in a neighborhood quite close to the steady-state. It turns out that both properties, history dependence and a varying speed of convergence, are generic with multiple state (i.e. "non-jumping") variables. The "proof" lies mainly in pointing out the necessity of an N-dimensional surface to span an N-dimensional space. That is, to assure that *some* steady state can be reached from any feasible starting-value combination of state variables, the dimensionality of possibly multiple steady states plus the dimensionality of the transition surface to each of these must sum to the number of state variables.

A steady state with dimensionality one or more is equivalent to history dependence. Such history dependence is more common than is often believed. Barro (1979) shows there is no one optimal level of government debt; rather, a country's optimal debt depends on its specific history of shocks (i.e. wars, famines, baby booms, natural resource finds, etc.). In two sector endogenous growth models (Mulligan and Sala-i-Martin, 1993; Caballe and Santos, 1993), the ratio but not the level of human to physical capital is determinate (the level however is less interesting within an endogenous growth framework).

The *linearization* of a transition path with two or more dimensions will always show an increasing speed of convergence: near the steady-state the negative eigenvalue lowest in absolute value will dominate; as one moves away from the steady-state, the negative eigenvalue highest in absolute value will dominate. The algebra showing this follows immediately below. The actual transition path (rather than its linearized approximation) may show an increasing, constant, or decreasing speed of convergence. A constant speed of convergence, however, would be a razor thin result.

The canonical underlying structural form for growth regressions is,

$$\frac{\mathrm{d}}{\mathrm{d}t}\log z = \frac{\mathrm{d}}{\mathrm{d}t}\left(\log z - \log z^*\right) = -\lambda \cdot \left(\log z - \log z^*\right) \tag{D.1}$$

Some definitions:

$$\mathbf{b} \equiv \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ \vdots \\ b_n \end{bmatrix} \qquad \mathbf{z} \equiv \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ \vdots \\ z_n \end{bmatrix} \qquad \mathbf{1} \equiv \left.\begin{bmatrix} 1 \\ 1 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}\right\} \text{n times}$$

$$\mathbf{b} \odot \mathbf{z} \equiv \begin{bmatrix} b_1 z_1 \\ b_2 z_2 \\ \vdots \\ \vdots \\ b_n z_n \end{bmatrix} \qquad \mathbf{b} \oslash \mathbf{z} \equiv \begin{bmatrix} b_1/z_1 \\ b_2/z_2 \\ \vdots \\ \vdots \\ b_n/z_n \end{bmatrix} \qquad \log \mathbf{z} \equiv \begin{bmatrix} \log z_1 \\ \log z_2 \\ \vdots \\ \vdots \\ \log z_n \end{bmatrix}$$

$$\mathbf{A}(\mathbf{z}) \equiv \begin{bmatrix} A_{11}(\mathbf{z}) & A_{12}(\mathbf{z}) & \cdots & \cdots & A_{1n}(\mathbf{z}) \\ A_{21}(\mathbf{z}) & \ddots & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ A_{n1}(\mathbf{z}) & \cdots & \cdots & \cdots & A_{nn}(\mathbf{z}) \end{bmatrix}$$

$$
\frac{\partial \mathbf{A}(\mathbf{z})}{\partial \mathbf{z}} \equiv \begin{bmatrix} \frac{\partial A_{11}(\mathbf{z})}{\partial z_1} & \frac{\partial A_{12}(\mathbf{z})}{\partial z_2} & \cdots & \cdots & \frac{\partial A_{1n}(\mathbf{z})}{\partial z_n} \\ \frac{\partial A_{21}(\mathbf{z})}{\partial z_1} & \ddots & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ \frac{\partial A_{n1}(\mathbf{z})}{\partial z_1} & \cdots & \cdots & \cdots & \frac{\partial A_{nn}(\mathbf{z})}{\partial z_n} \end{bmatrix}
$$

Suppose a system of differential equations defined in terms of the logarithms of the vector of variables, $\mathbf{z}$:

$$
\frac{\mathrm{d}}{\mathrm{d}t} \log \mathbf{z} = \mathbf{A}(\log \mathbf{z}) \tag{D.2}
$$

Take a Taylor expansion around the system's steady-state:

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t} \log \mathbf{z} &\approx \mathbf{A}(\log \mathbf{z}^*) + \left.\frac{\partial \mathbf{A}(\log \mathbf{z})}{\partial \log \mathbf{z}}\right|_{\log \mathbf{z} = \log \mathbf{z}^*} \cdot (\log \mathbf{z} - \log \mathbf{z}^*) && \text{(D.3a)} \\
&= \mathbf{J} \, \log(\mathbf{z} \oslash \mathbf{z}^*) && \text{(D.3b)}
\end{aligned}
$$

$$
\mathbf{J} \equiv \left.\frac{\partial \mathbf{A}(\log \mathbf{z})}{\partial \log \mathbf{z}}\right|_{\log \mathbf{z} = \log \mathbf{z}^*}
$$

Let $|\lambda_f| > |\lambda_s|$ represent two *negative* eignevalues of $\mathbf{J}$ with corresponding eigenvectors $\mathbf{v}_f$ and $\mathbf{v}_s$, and arbitrary weighting scalars, a_f and a_s ("f" is meant to connote "fast" and "s", "slow").

$$
\mathbf{J} \; (a_f \mathbf{v}_f + a_s \mathbf{v}_s) = -(\lambda_f a_f \mathbf{v}_f + \lambda_s a_s \mathbf{v}_s) \tag{D.4}
$$

Then the solution to (D.3b) can be written as,

$$
\log(\mathbf{z} \oslash \mathbf{z}^*) \approx a_f \mathbf{v}_f \mathrm{e}^{-\lambda_f t} + a_s \mathbf{v}_s \mathrm{e}^{-\lambda_s t} \tag{D.5}
$$

Take the derivative of (D.5) with respect to t,

$$
\frac{\mathrm{d}}{\mathrm{d}t} \log(\mathbf{z} \oslash \mathbf{z}^*) \approx -\left(\lambda_f a_f \mathbf{v}_f \mathrm{e}^{-\lambda_f t} + \lambda_s a_s \mathbf{v}_s \mathrm{e}^{-\lambda_s t}\right) \tag{D.6}
$$

For the vector analog to (D.1) we want,

$$
\boldsymbol{\lambda} \quad \text{s.t.} \quad \frac{\mathrm{d}}{\mathrm{d}t} \log(\mathbf{z} \oslash \mathbf{z}^*) = -\boldsymbol{\lambda} \odot \log(\mathbf{z} \oslash \mathbf{z}^*)
$$

Substituting using (D.5) and (D.6) gives,

$$
\boldsymbol{\lambda} \quad \text{s.t.} \quad \left(\lambda_f a_f \mathbf{v}_f \mathrm{e}^{-\lambda_f t} + \lambda_s a_s \mathbf{v}_s \mathrm{e}^{-\lambda_s t}\right) \approx \boldsymbol{\lambda} \odot \left(a_f \mathbf{v}_f \mathrm{e}^{-\lambda_f t} + a_s \mathbf{v}_s \mathrm{e}^{-\lambda_s t}\right) \tag{D.7}
$$

It is immediately evident that unless either a_f or a_s equal zero, $\boldsymbol{\lambda}$ will differ in its elements *and* vary with time. I now formally define $\boldsymbol{\lambda}(t)$ as,

$$
\boldsymbol{\lambda}(t) \equiv -\frac{\mathrm{d}}{\mathrm{d}t} \log(\mathbf{z} \oslash \mathbf{z}^*) \oslash \log(\mathbf{z} \oslash \mathbf{z}^*)
$$

The log linearization therefore approximates the speed of convergence as,

$$\boldsymbol{\lambda}(t) \approx \left(\lambda_f a_f \mathbf{v}_f \mathrm{e}^{-\lambda_f t} + \lambda_s a_s \mathbf{v}_s \mathrm{e}^{-\lambda_s t}\right) \oslash \left(a_f \mathbf{v}_f \mathrm{e}^{-\lambda_f t} + a_s \mathbf{v}_s \mathrm{e}^{-\lambda_s t}\right) \tag{D.8}$$

Normalize the eigenvectors so that the first element of each equals one. Then the speed of convergence corresponding to this first element is given by,

$$\lambda_1(t) \approx \frac{\lambda_f a_f \mathrm{e}^{-\lambda_f t} + \lambda_s a_s \mathrm{e}^{-\lambda_s t}}{a_f \mathrm{e}^{-\lambda_f t} + a_s \mathrm{e}^{-\lambda_s t}} \tag{D.9}$$

So except in the special case when a_f or a_s equal zero, the linearization implies the speed of convergence for this representative first element will go from a_f to a_s as time goes from negative to positive infinity. If a_f and a_s are oppositely signed, it will also asymptote to positive and negative infinity at some intermediate time.

Bibliography

Ades, Alberto F. and Edward L. Glaeser (1995). "Trade and Circuses: Explaining Urban Giants." *Quarterly Journal of Economics* 110, 1 (February), pp. 195 - 228.

Barro, Robert J. (1979). "On the Determination of Public Debt." *Journal of Political Economy*, 87, (Oct.), pp. 940 – 971.

Barro, Robert J. and Xavier Sala-i-Martin, (1991). "Convergence across States and Regions." *Brookings Papers on Economic Activity*, 1:1991, pp. 107 – 182.

Barro, Robert J. and Xavier Sala-i-Martin (1995). *Economic Growth*. New York: McGraw Hill.

Blanchard, Olivier J. and Lawrence F. Katz (1992). "Regional Evolutions." *Brookings Papers on Economic Activity* 1:1992.

Blanchard, Olivier, Changyong Rhee, and Lawrence Summers (1993). "The Stock Market, Profit, and Investment." *Quarterly Journal of Economics* 108, 1 (Feb.), pp. 115 – 136.

Borts, George H. (1960). "Regional Cycles of Manufacturing Employment in the United States." *Jorurnal of the American Statistical Association* 55, pp. 151-211. Reprinted as NBER Occasional Paper 73.

Braun, Juan (1993). *Essays on Economic Growth and Migration*. Ph.D. dissertation, Harvard University.

Caballé, Jordi and Manuel S. Santos (1993). "On Endogenous Growth with Physical and Human Capital." *Journal of Political Economy* 101, 6 (Dec.), pp. 1042 – 1067.

Caselli, Francesco, Gerardo Esquivel, and Fernando LefortCaselli, Francesco, Gerardo Esquivel, and Fernando Lefort (1996). "Reopening the Convergence Debate: A New Look at Cross-Country Growth Empirics." *Journal of Economic Growth* 1, 3 (September), pp. 363-389.

Glaeser, Edward L., José A. Scheinkman, and Andrei Shleifer (1995). "Economic Growth in a Cross-Section of Cities." *Journal of Monetary Economics* 36, pp. 117 – 143.

Greenwood, Michael J.; Gary L. Hunt; Dan S. Rickman; George I. Treyz (1991). "Migration, Regional Equilibrium, and the Estimation of Compensating Differentials." *American Economic Review* 81, 5 (Dec.), pp. 1382 –1390.

Gyourko, Joseph and Joseph Tracy (1989). "The Importance of Local Fiscal Conditions in Analyzing Local Labor Markets." *Journal of Political Economy* 97, 5 (Oct.), pp. 1208 – 1231.

Gyourko, Joseph and Joseph Tracy (1991). "The Structure of Local Public Finance and the Quality of Life." *Journal of Political Economy* 99, 4 (Aug.), pp. 774 – 806.

Hayashi, Fumio (1982). "Tobin's Marginal q and Average q: A Neoclassical Interpretation." *Econometrica* 50, 1 (Jan.), pp. 213 – 224..

Islam, Nazrul (1995). "Growth Empirics: a Panel Data Approach." *Quarterly Journal of Economics* 110, 4 (November), pp. 1127-1170.

King, Robert G. and Sergio T. Rebelo (1993). "Transitional Dynamics and Economic Growth in the Neoclassical Model." *American Economic Review* 83, 4 (September), pp. 908 - 931.

Koopmans, Tjalling C. (1965). "On the Concept of Optimal Economic Growth." *The Econometric Approach to Development Planning.* Amsterdam: North Holland.

Krugman, Paul (1991). *Geography and Trade.* Cambridge, MA: MIT Press.

Mueser, Peter R. and Philip E. Graves (1995). "Examining the Role of Economic Opportunity and Amenities in Explaining Population Redistribution." *Journal of Urban Economics* 37, 2 (March), pp. 176 – 200.

Mulligan, Casey B. and Xavier Sala-i-Martin (1993). "Transitional Dynamics in Two-Sector Models of Endogenous Growth." *Quarterly Journal of Economics* 108, 3 (Aug) pp. 739 – 773.

Ramsey, Frank (1928). "A Mathematical Theory of Saving." *Economic Journal* 38 (Dec.), pp. 543 – 559.

Rappaport, Jordan (1998). "How the Speed of Convergence Varies." Mimeo, Center for International Development, Harvard University, (December).

Rappaport, Jordan (1999). "Local Growth Empirics." Mimeo, Center for International Development, Harvard University, (May).

Rappaport, Jordan and Jeffrey Sachs (1999). "The U.S. as a Coastal Nation." Mimeo, Center for International Development at Harvard University, (May).

Rappaport, Jordan and Jeffrey Sachs (1998). "Cross-Sectional Data on Geography, Transportation Infrastructure, and Civic Culture for U.S. Counties Circa 1970." Working Version, Center for International Development at Harvard University, (November).

Roback, Jennifer (1982). "Wages, Rents, and the Quality of Life." *Journal of Political Economy* 90, 6 (Dec.), pp. 1257 – 1278.

Rosen, Sherwin (1979). "Wage-Based Indexes of Urban Quality of Life." In Miezkowski and Straszheim, Eds., *Current Issues in Urban Economics.* Baltimore: Johns Hopkins University Press.

Tiebout, Charles M. (1956). "A Pure Theory of Local Expenditures." *Journal of Political Economy* 64, 5 (Oct.), pp. 416 – 424.

Table 2: Mobility and Income Convergence

Numerical results for a negative capital shock that drops wages to 60% of their steady-state level. For narrow capital share, (α = 0.30), per capita capital stock drops to 18.2% of its steady-state level.

A. High Capital Mobility (q_K* = 1.14), Narrow Capital Share (α = 0.30)

(1) labor mobility	(2) labor friction (ω)	(3) Initial Growth Rates (at t= 0) pop	(4) capital	(5) wages	(6) Population Minimum level	(7) time	(8) Relative Wage at Time (in years) t=1	(9) t=5	(10) t=10	(11) t=20	(12) Time to Reach Relative Wage 80%	(13) 90%	(14) 95%	(15) 99%
	1.000625	-0.212	0.397	0.182	0.72	4.3	0.697	0.888	0.952	0.985	2.6	5.6	9.7	24.0
high	1.00125	-0.132	0.426	0.167	0.78	5.1	0.689	0.878	0.952	0.987	2.8	6.0	9.7	22.9
	1.0025	-0.078	0.443	0.156	0.84	6.2	0.684	0.869	0.951	0.988	3.0	6.3	9.9	21.4
	1.005	-0.044	0.461	0.152	0.89	7.2	0.681	0.863	0.948	0.990	3.2	6.6	10.2	20.2
base	1.01	-0.025	0.476	0.150	0.93	8.1	0.679	0.859	0.946	0.990	3.3	6.8	10.5	19.8
	1.02	-0.013	0.478	0.147	0.96	9.0	0.679	0.856	0.944	0.990	3.3	6.9	10.6	19.7
	1.04	-0.007	0.489	0.149	0.98	9.6	0.679	0.855	0.943	0.990	3.3	7.0	10.7	19.8
low	1.08	-0.004	0.485	0.147	0.99	9.8	0.678	0.855	0.942	0.990	3.3	7.0	10.8	19.8
	1.16	-0.002	0.487	0.147	0.99	9.7	0.678	0.854	0.942	0.990	3.4	7.0	10.8	19.9
	1.32	-0.001	0.489	0.147	1.00	9.4	0.678	0.854	0.942	0.990	3.4	7.0	10.8	19.9
zero	∞		0.483	0.145			0.678	0.854	0.942	0.990	3.4	7.0	10.9	19.9

B. Base Capital Mobility (q_K* = 1.56), Narrow Capital Share (α = 0.30)

(1) labor mobility	(2) labor friction (ω)	(3) Initial Growth Rates (at t= 0) pop	(4) capital	(5) wages	(6) Population Minimum level	(7) time	(8) Relative Wage at Time (in years) t=1	(9) t=5	(10) t=10	(11) t=20	(12) Time to Reach Relative Wage 80%	(13) 90%	(14) 95%	(15) 99%
	1.000625	-0.246	0.193	0.132	0.63	5.5	0.673	0.834	0.903	0.951	3.7	9.6	19.7	47.7
high	1.00125	-0.159	0.204	0.109	0.68	6.9	0.661	0.818	0.899	0.953	4.4	10.1	19.0	47.4
	1.0025	-0.100	0.217	0.095	0.74	8.4	0.654	0.802	0.892	0.955	4.9	10.7	18.5	46.4
	1.005	-0.061	0.229	0.087	0.80	10.0	0.649	0.789	0.883	0.956	5.4	11.4	18.5	44.1
base	1.01	-0.035	0.240	0.083	0.86	12.0	0.647	0.781	0.876	0.955	5.8	12.0	18.9	40.7
	1.02	-0.020	0.248	0.080	0.91	13.6	0.646	0.776	0.870	0.953	6.0	12.5	19.4	38.4
	1.04	-0.011	0.253	0.079	0.94	15.2	0.645	0.774	0.867	0.951	6.1	12.8	19.8	37.5
low	1.08	-0.006	0.259	0.080	0.97	16.1	0.645	0.772	0.865	0.950	6.2	13.0	20.0	37.3
	1.16	-0.003	0.260	0.079	0.98	16.9	0.645	0.772	0.864	0.949	6.3	13.1	20.2	37.3
	1.32	-0.002	0.263	0.079	0.99	17.1	0.645	0.772	0.864	0.949	6.3	13.1	20.3	37.3
zero	∞		0.259	0.078			0.645	0.771	0.863	0.948	6.3	13.2	20.4	37.4

C. Low Capital Mobility (q_K* = 3.24), Narrow Capital Share (α = 0.30)

(1) labor mobility	(2) labor friction (ω)	(3) Initial Growth Rates (at t= 0) pop	(4) capital	(5) wages	(6) Population Minimum level	(7) time	(8) Relative Wage at Time (in years) t=1	(9) t=5	(10) t=10	(11) t=20	(12) Time to Reach Relative Wage 80%	(13) 90%	(14) 95%	(15) 99%
	1.000625	-0.260	0.115	0.113	0.57	6.5	0.662	0.804	0.867	0.916	4.8	15.7	33.3	75.5
high	1.00125	-0.174	0.122	0.089	0.62	8.2	0.650	0.784	0.861	0.918	5.7	15.6	32.6	76.0
	1.0025	-0.111	0.128	0.072	0.67	10.0	0.642	0.765	0.849	0.919	6.7	16.0	31.3	76.3
	1.005	-0.071	0.137	0.062	0.73	12.3	0.636	0.749	0.836	0.917	7.6	16.9	30.1	76.2
base	1.01	-0.043	0.145	0.057	0.79	15.0	0.633	0.737	0.823	0.913	8.4	17.9	29.6	74.0
	1.02	-0.025	0.151	0.053	0.85	17.6	0.631	0.730	0.814	0.908	9.0	18.8	29.9	68.1
	1.04	-0.014	0.156	0.051	0.90	20.6	0.630	0.726	0.808	0.903	9.4	19.6	30.6	62.9
low	1.08	-0.008	0.160	0.050	0.94	22.7	0.629	0.724	0.805	0.899	9.6	20.1	31.1	60.3
	1.16	-0.004	0.165	0.051	0.96	24.6	0.629	0.723	0.803	0.897	9.8	20.4	31.5	59.4
	1.32	-0.002	0.165	0.050	0.98	25.3	0.629	0.722	0.802	0.896	9.8	20.6	31.8	59.2
zero	∞		0.167	0.050			0.629	0.721	0.801	0.895	9.9	20.8	32.1	59.1

Table 2: Mobility and Income Convergence (continued)

Numerical results for a negative capital shock that drops wages to 60% of their steady-state level. For narrow capital share, (α = 0.60), per capita capital stock drops to 42.7% of its steady-state level.

D. High Capital Mobility (q_K* = 1.14), Broad Capital Share (α = 0.60)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
labor mobility	labor friction	Initial Growth Rates (at t= 0)			Population Minimum		Relative Wage at Time (in years)				Time to Reach Relative Wage			
	(ω)	pop	capital	wages	level	time	t=1	t=5	t=10	t=20	80%	90%	95%	99%
	1.000625	-0.179	0.095	0.164	0.74	4.9	0.690	0.871	0.930	0.967	2.8	6.7	14.1	38.1
high	1.00125	-0.118	0.102	0.132	0.78	5.9	0.674	0.851	0.928	0.969	3.4	7.5	13.7	37.5
	1.0025	-0.072	0.108	0.108	0.82	7.1	0.663	0.831	0.921	0.972	4.0	8.3	13.7	36.0
	1.005	-0.044	0.115	0.095	0.87	8.7	0.655	0.814	0.912	0.974	4.5	9.1	14.2	33.1
base	1.01	-0.025	0.120	0.087	0.91	10.1	0.651	0.802	0.903	0.974	4.9	9.8	14.8	30.1
	1.02	-0.014	0.124	0.083	0.94	11.3	0.649	0.795	0.896	0.973	5.2	10.3	15.4	28.8
	1.04	-0.008	0.128	0.081	0.96	12.5	0.648	0.790	0.892	0.971	5.4	10.6	15.8	28.5
low	1.08	-0.004	0.131	0.081	0.98	13.1	0.647	0.788	0.890	0.970	5.4	10.8	16.0	28.5
	1.16	-0.002	0.130	0.080	0.99	13.2	0.647	0.787	0.888	0.970	5.5	10.9	16.2	28.6
	1.32	-0.001	0.132	0.080	0.99	13.3	0.647	0.786	0.888	0.969	5.5	10.9	16.2	28.6
zero	∞		0.128	0.077			0.646	0.786	0.887	0.969	5.5	11.0	16.3	28.8

E. Base Capital Mobility (q_K* = 1.56), Broad Capital Share (α = 0.60)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
labor mobility	labor friction	Initial Growth Rates (at t= 0)			Population Minimum		Relative Wage at Time (in years)				Time to Reach Relative Wage			
	(ω)	pop	capital	wages	level	time	t=1	t=5	t=10	t=20	80%	90%	95%	99%
	1.000625	-0.192	0.045	0.142	0.68	6.1	0.678	0.836	0.890	0.927	3.6	12.0	30.6	69.9
high	1.00125	-0.129	0.048	0.106	0.71	7.7	0.660	0.811	0.883	0.929	4.6	12.5	29.9	71.0
	1.0025	-0.086	0.051	0.082	0.75	9.5	0.647	0.784	0.869	0.930	5.7	13.5	28.5	71.8
	1.005	-0.054	0.055	0.066	0.79	11.7	0.638	0.760	0.851	0.928	6.9	14.9	27.3	72.4
base	1.01	-0.034	0.059	0.055	0.84	14.2	0.633	0.741	0.832	0.923	8.0	16.4	27.1	71.4
	1.02	-0.020	0.062	0.049	0.88	17.1	0.629	0.728	0.817	0.915	8.9	17.8	27.8	66.4
	1.04	-0.011	0.065	0.046	0.92	19.5	0.627	0.720	0.806	0.907	9.6	19.0	28.8	59.3
low	1.08	-0.006	0.067	0.044	0.95	21.2	0.626	0.716	0.800	0.902	10.0	19.8	29.7	56.0
	1.16	-0.003	0.068	0.043	0.97	22.9	0.626	0.714	0.796	0.898	10.3	20.3	30.3	55.0
	1.32	-0.002	0.068	0.042	0.98	23.6	0.625	0.712	0.795	0.896	10.4	20.5	30.6	54.8
zero	∞		0.068	0.041			0.625	0.711	0.792	0.894	10.6	20.9	31.1	54.8

F. Low Capital Mobility (q_K* = 3.24), Broad Capital Share (α = 0.60)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
labor mobility	labor friction	Initial Growth Rates (at t= 0)			Population Minimum		Relative Wage at Time (in years)				Time to Reach Relative Wage			
	(ω)	pop	capital	wages	level	time	t=1	t=5	t=10	t=20	80%	90%	95%	99%
	1.000625	-0.191	0.026	0.130	0.66	7.3	0.672	0.816	0.864	0.896	4.2	21.8	53.3	98.3
high	1.00125	-0.129	0.028	0.094	0.68	9.2	0.654	0.789	0.855	0.896	5.5	21.5	53.2	102.5
	1.0025	-0.087	0.029	0.070	0.71	11.5	0.641	0.760	0.838	0.896	7.1	21.5	52.2	105.3
	1.005	-0.058	0.032	0.054	0.75	14.4	0.632	0.733	0.815	0.890	8.9	22.4	50.2	108.2
base	1.01	-0.037	0.034	0.042	0.79	17.9	0.625	0.712	0.791	0.879	10.7	24.2	47.2	111.0
	1.02	-0.022	0.035	0.035	0.83	21.6	0.621	0.696	0.770	0.865	12.5	26.4	45.3	112.1
	1.04	-0.013	0.038	0.031	0.88	25.6	0.619	0.686	0.755	0.851	14.1	28.5	45.5	108.6
low	1.08	-0.008	0.040	0.029	0.92	30.0	0.617	0.680	0.746	0.840	15.2	30.3	46.5	100.3
	1.16	-0.004	0.041	0.028	0.95	33.0	0.617	0.677	0.740	0.833	15.9	31.6	47.7	92.5
	1.32	-0.002	0.042	0.027	0.97	36.1	0.616	0.675	0.737	0.829	16.4	32.4	48.6	84.6
zero	∞		0.043	0.026			0.616	0.673	0.733	0.824	17.0	33.6	50.0	87.9

Table 3: Mobility Following a Productivity Shock

Numerical results for a positive change in total factor productivity such that new steady-state wage level is 1.05 times old level. For narrow capital share, (α = 0.30), this implies a 3.47% rise in TFP.

A. High Capital Mobility (q_K* = 1.14), Narrow Capital Share (α = 0.30)

(1) labor mobility	(2) labor friction (ω)	(3) Initial Growth Rates (at t= 0) capital	(4) wages	(5) house val	(6) House Value, Asset Wealth, Population (Initial & Steady State Levels) init. house	(7) s.s. house	(8) s.s. assts	(9) s.s. pop	(10) Net Migration Rate at Time (in years) t=0	(11) t=10	(12) t=20	(13) Time to Close Dist. to S.S. 50%	(14) 75%
	1.0025	0.0186	0.0009	0.0027	1.209	1.250	0.968	1.199	0.0156	0.0065	0.0028	8.1	16.3
	1.005	0.0168	0.0016	0.0026	1.199	1.250	0.971	1.198	0.0115	0.0063	0.0032	10.6	21.1
base	1.01	0.0149	0.0020	0.0024	1.187	1.250	0.973	1.198	0.0082	0.0055	0.0034	14.5	28.7
	1.02	0.0132	0.0022	0.0022	1.170	1.249	0.977	1.198	0.0057	0.0044	0.0032	20.7	41.0
	1.04	0.0118	0.0024	0.0018	1.150	1.249	0.979	1.197	0.0038	0.0033	0.0026	30.8	61.2
low	1.08	0.0107	0.0025	0.0015	1.127	1.249	0.981	1.197	0.0025	0.0023	0.0020	47.6	95.0
	1.16	0.0099	0.0025	0.0011	1.105	1.249	0.982	1.197	0.0016	0.0015	0.0014	76.4	145.3
	1.32	0.0093	0.0025	0.0008	1.086	1.249	0.983	1.197	0.0010	0.0009	0.0009	124.9	205.0

B. Base Capital Mobility (q_K* = 1.56), Narrow Capital Share (α = 0.30)

(1) labor mobility	(2) labor friction (ω)	(3) Initial Growth Rates (at t= 0) capital	(4) wages	(5) house val	(6) House Value, Asset Wealth, Population (Initial & Steady State Levels) init. house	(7) s.s. house	(8) s.s. assts	(9) s.s. pop	(10) Net Migration Rate at Time (in years) t=0	(11) t=10	(12) t=20	(13) Time to Close Dist. to S.S. 50%	(14) 75%
	1.005	0.0102	0.0001	0.0025	1.187	1.248	0.948	1.200	0.0100	0.0055	0.0032	13.2	27.4
base	1.01	0.0092	0.0006	0.0023	1.176	1.248	0.953	1.199	0.0073	0.0049	0.0033	17.1	34.4
	1.02	0.0082	0.0009	0.0021	1.162	1.247	0.958	1.198	0.0052	0.0040	0.0030	23.3	46.1
	1.04	0.0072	0.0011	0.0018	1.143	1.247	0.962	1.198	0.0035	0.0031	0.0025	33.2	65.6
low	1.08	0.0064	0.0012	0.0014	1.123	1.247	0.967	1.197	0.0023	0.0022	0.0019	49.9	98.8
	1.16	0.0058	0.0013	0.0011	1.102	1.247	0.970	1.197	0.0015	0.0014	0.0013	78.2	155.5
	1.32	0.0053	0.0013	0.0008	1.083	1.247	0.972	1.197	0.0009	0.0009	0.0009	126.4	239.9

C. Low Capital Mobility (q_K* = 3.24), Narrow Capital Share (α = 0.30)

(1) labor mobility	(2) labor friction (ω)	(3) Initial Growth Rates (at t= 0) capital	(4) wages	(5) house val	(6) House Value, Asset Wealth, Population (Initial & Steady State Levels) init. house	(7) s.s. house	(8) s.s. assts	(9) s.s. pop	(10) Net Migration Rate at Time (in years) t=0	(11) t=10	(12) t=20	(13) Time to Close Dist. to S.S. 50%	(14) 75%
	1.02	0.0056	0.0003	0.0019	1.150	1.243	0.945	1.198	0.0047	0.0036	0.0028	26.9	54.3
	1.04	0.0050	0.0005	0.0017	1.135	1.243	0.950	1.197	0.0033	0.0028	0.0024	36.9	73.2
low	1.08	0.0044	0.0007	0.0014	1.116	1.243	0.956	1.196	0.0022	0.0020	0.0018	53.6	105.6
	1.16	0.0039	0.0008	0.0010	1.097	1.243	0.961	1.195	0.0014	0.0013	0.0013	81.8	161.8
	1.32	0.0035	0.0008	0.0007	1.080	1.243	0.964	1.195	0.0009	0.0009	0.0008	129.6	257.3

Table 3: Mobility Following a Productivity Shock (continued)

Numerical results for a positive change in total factor productivity such that new steady-state wage level is 1.05 times old level. For broad capital share, (α = 0.60), this implies a 1.97% rise in TFP.

D. High Capital Mobility (q_K* = 1.14), Broad Capital Share (α = 0.60)

(1) labor mobility	(2) labor friction (ω)	(3) Initial Growth Rates (at t= 0): capital	(4) wages	(5) house val	(6) House Value, Asset Wealth, Population (Initial & Steady State Levels): init. house	(7) s.s. house	(8) s.s. assts	(9) s.s. pop	(10) Net Migration Rate at Time (in years): t=0	(11) t=10	(12) t=20	(13) Time to Close Dist. to S.S.: 50%	(14) 75%
high	1.00125	0.0127	0.0012	0.0022	1.160	1.201	0.978	1.164	0.0107	0.0050	0.0026	10.5	21.4
	1.0025	0.0121	0.0020	0.0021	1.156	1.201	0.979	1.164	0.0087	0.0050	0.0028	11.8	23.6
	1.005	0.0113	0.0026	0.0021	1.151	1.201	0.980	1.164	0.0069	0.0047	0.0029	14.0	27.5
base	1.01	0.0105	0.0031	0.0019	1.143	1.200	0.981	1.163	0.0053	0.0042	0.0029	17.6	34.2
	1.02	0.0095	0.0034	0.0017	1.131	1.200	0.983	1.162	0.0039	0.0034	0.0026	23.5	45.7
	1.04	0.0086	0.0035	0.0015	1.116	1.200	0.984	1.162	0.0027	0.0026	0.0021	33.3	65.2
low	1.08	0.0080	0.0037	0.0012	1.099	1.200	0.986	1.162	0.0018	0.0018	0.0016	49.8	98.4
	1.16	0.0073	0.0037	0.0009	1.082	1.200	0.986	1.161	0.0012	0.0012	0.0011	78.1	155.4
	1.32	0.0070	0.0038	0.0006	1.067	1.200	0.987	1.161	0.0007	0.0008	0.0007	126.8	224.3

E. Base Capital Mobility (q_K* = 1.56), Broad Capital Share (α = 0.60)

(1) labor mobility	(2) labor friction (ω)	(3) Initial Growth Rates (at t= 0): capital	(4) wages	(5) house val	(6) House Value, Asset Wealth, Population (Initial & Steady State Levels): init. house	(7) s.s. house	(8) s.s. assts	(9) s.s. pop	(10) Net Migration Rate at Time (in years): t=0	(11) t=10	(12) t=20	(13) Time to Close Dist. to S.S.: 50%	(14) 75%
	1.005	0.0063	0.0007	0.0018	1.129	1.196	0.968	1.164	0.0052	0.0035	0.0025	21.3	43.5
base	1.01	0.0059	0.0011	0.0017	1.124	1.195	0.969	1.163	0.0041	0.0032	0.0025	24.7	49.2
	1.02	0.0054	0.0014	0.0016	1.115	1.195	0.972	1.163	0.0031	0.0027	0.0023	30.4	59.4
	1.04	0.0050	0.0017	0.0014	1.104	1.195	0.974	1.162	0.0023	0.0021	0.0019	40.0	77.6
low	1.08	0.0045	0.0018	0.0011	1.090	1.195	0.976	1.161	0.0016	0.0016	0.0015	56.2	109.6
	1.16	0.0041	0.0019	0.0008	1.075	1.195	0.978	1.160	0.0010	0.0011	0.0010	84.2	165.3
	1.32	0.0039	0.0019	0.0006	1.062	1.195	0.979	1.160	0.0007	0.0007	0.0007	131.7	260.6

F. Low Capital Mobility (q_K* = 3.24), Broad Capital Share (α = 0.60)

(1) labor mobility	(2) labor friction (ω)	(3) Initial Growth Rates (at t= 0): capital	(4) wages	(5) house val	(6) House Value, Asset Wealth, Population (Initial & Steady State Levels): init. house	(7) s.s. house	(8) s.s. assts	(9) s.s. pop	(10) Net Migration Rate at Time (in years): t=0	(11) t=10	(12) t=20	(13) Time to Close Dist. to S.S.: 50%	(14) 75%
base	1.01	0.0037	0.0003	0.0014	1.102	1.186	0.963	1.160	0.0033	0.0025	0.0019	34.6	71.1
	1.02	0.0035	0.0006	0.0013	1.097	1.186	0.965	1.159	0.0025	0.0021	0.0018	40.2	80.3
	1.04	0.0032	0.0008	0.0012	1.088	1.186	0.968	1.158	0.0018	0.0017	0.0016	49.8	97.4
low	1.08	0.0029	0.0010	0.0010	1.078	1.186	0.971	1.157	0.0013	0.0013	0.0012	66.2	128.2
	1.16	0.0027	0.0011	0.0008	1.066	1.186	0.973	1.156	0.0009	0.0009	0.0009	94.1	182.8
	1.32	0.0025	0.0012	0.0005	1.055	1.185	0.975	1.155	0.0006	0.0006	0.0006	141.5	277.2

Table 4: Mobility Following a Quality-of-Life Shock

Numerical results for a positive change in quality of life such that individuals are willing to pay 20% more for housing services while still attaining their reservation level of utility.

A. High Capital Mobility (q_K* = 1.14), Narrow Capital Share (α = 0.30)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
labor mobility	labor friction	Initial Growth Rates (at t= 0)			Relative Wage Minimum		House Value, Assets, Pop. (Initial, Steady State Levels)			Net Migration Rate at Time (in years)			Time to Close Dist. to S.S.	
	(ω)	capital	wages	house val	level	time	init. house	s.s. assts	s.s. pop	t=0	t=10	t=20	50%	75%
	1.000625	0.0151	-0.0064	0.0030	0.990	4.5	1.174	0.975	1.203	0.0363	0.0051	0.0017	4.1	9.8
high	1.00125	0.0132	-0.0038	0.0029	0.992	5.6	1.169	0.978	1.202	0.0259	0.0059	0.0020	5.4	11.8
	1.0025	0.0111	-0.0023	0.0028	0.994	6.6	1.163	0.982	1.202	0.0187	0.0064	0.0026	7.1	14.9
	1.005	0.0089	-0.0013	0.0026	0.996	8.2	1.153	0.985	1.202	0.0133	0.0063	0.0031	9.7	19.9

B. Base Capital Mobility (q_K* = 1.56), Narrow Capital Share (α = 0.30)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
labor mobility	labor friction	Initial Growth Rates (at t= 0)			Relative Wage Minimum		House Value, Assets, Pop. (Initial, Steady State Levels)			Net Migration Rate at Time (in years)			Time to Close Dist. to S.S.	
	(ω)	capital	wages	house val	level	time	init. house	s.s. assts	s.s. pop	t=0	t=10	t=20	50%	75%
	1.000625	0.0092	-0.0071	0.0028	0.986	5.8	1.162	0.956	1.205	0.0329	0.0043	0.0021	5.2	15.4
high	1.00125	0.0084	-0.0047	0.0027	0.988	7.2	1.159	0.960	1.205	0.0241	0.0051	0.0023	6.6	17.0
	1.0025	0.0075	-0.0030	0.0026	0.990	9.1	1.154	0.964	1.204	0.0173	0.0057	0.0026	8.5	19.7
	1.005	0.0063	-0.0019	0.0025	0.992	11.0	1.146	0.969	1.204	0.0126	0.0057	0.0030	11.2	24.3
base	1.01	0.0051	-0.0011	0.0024	0.994	13.3	1.135	0.975	1.203	0.0089	0.0052	0.0032	15.2	31.8
	1.02	0.0040	-0.0007	0.0021	0.996	16.4	1.119	0.981	1.202	0.0061	0.0043	0.0031	21.5	43.9
	1.04	0.0024	-0.0009	0.0018	0.993	22.9	1.094	0.988	1.204	0.0038	0.0030	0.0024	35.9	60.4
low	1.08	0.0017	-0.0005	0.0014	0.995	28.3	1.075	0.992	1.203	0.0025	0.0022	0.0019	52.8	88.1

C. Low Capital Mobility (q_K* = 3.24), Narrow Capital Share (α = 0.30)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
labor mobility	labor friction	Initial Growth Rates (at t= 0)			Relative Wage Minimum		House Value, Assets, Pop. (Initial, Steady State Levels)			Net Migration Rate at Time (in years)			Time to Close Dist. to S.S.	
	(ω)	capital	wages	house val	level	time	init. house	s.s. assts	s.s. pop	t=0	t=10	t=20	50%	75%
	1.000625	0.0061	-0.0079	0.0027	0.983	6.7	1.150	0.944	1.208	0.0323	0.0035	0.0020	6.4	23.4
high	1.00125	0.0057	-0.0053	0.0026	0.985	8.8	1.147	0.947	1.207	0.0234	0.0043	0.0021	7.9	24.5
	1.0025	0.0052	-0.0034	0.0025	0.987	10.8	1.143	0.952	1.207	0.0166	0.0050	0.0024	9.9	26.6
	1.005	0.0046	-0.0022	0.0024	0.989	13.8	1.137	0.957	1.206	0.0120	0.0052	0.0028	12.8	30.7
base	1.01	0.0039	-0.0014	0.0023	0.992	16.7	1.127	0.964	1.205	0.0085	0.0049	0.0030	17.1	37.8
	1.02	0.0031	-0.0009	0.0021	0.994	21.1	1.114	0.971	1.204	0.0059	0.0041	0.0029	23.5	49.6

Table 4: Mobility Following a Quality-of-Life Shock (continued)

Numerical results for a positive change in quality of life such that individuals are willing to pay 20% more for housing services while still attaining their reservation level of utility.

D. High Capital Mobility (q_K* = 1.14), Broad Capital Share (α = 0.60)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
labor mobility	labor friction	Initial Growth Rates (at t= 0)			Relative Wage Minimum		House Value, Assets, Pop. (Initial, Steady State Levels)			Net Migration Rate at Time (in years)			Time to Close Dist. to S.S.	
	(ω)	capital	wages	house val	level	time	init. house	s.s. assts	s.s. pop	t=0	t=10	t=20	50%	75%
	1.000625	0.0108	-0.0125	0.0028	0.980	5.0	1.162	0.983	1.205	0.0316	0.0049	0.0024	6.0	15.9
high	1.00125	0.0098	-0.0082	0.0028	0.983	6.2	1.159	0.984	1.205	0.0235	0.0054	0.0026	7.2	17.3
	1.0025	0.0086	-0.0052	0.0027	0.986	7.5	1.154	0.986	1.204	0.0173	0.0058	0.0028	8.8	19.8
	1.005	0.0073	-0.0031	0.0026	0.990	9.4	1.146	0.988	1.204	0.0125	0.0058	0.0031	11.4	24.2
base	1.01	0.0058	-0.0018	0.0024	0.993	11.3	1.135	0.990	1.203	0.0088	0.0052	0.0033	15.3	31.4
	1.02	0.0044	-0.0010	0.0022	0.995	13.4	1.120	0.993	1.202	0.0061	0.0043	0.0031	21.5	43.4
	1.04	0.0031	-0.0006	0.0019	0.997	15.8	1.101	0.995	1.202	0.0041	0.0033	0.0026	31.6	63.4
low	1.08	0.0021	-0.0003	0.0015	0.998	19.1	1.080	0.997	1.201	0.0026	0.0023	0.0020	48.4	97.0
	1.16	0.0014	-0.0002	0.0011	0.999	22.0	1.059	0.998	1.201	0.0017	0.0015	0.0014	76.9	153.9

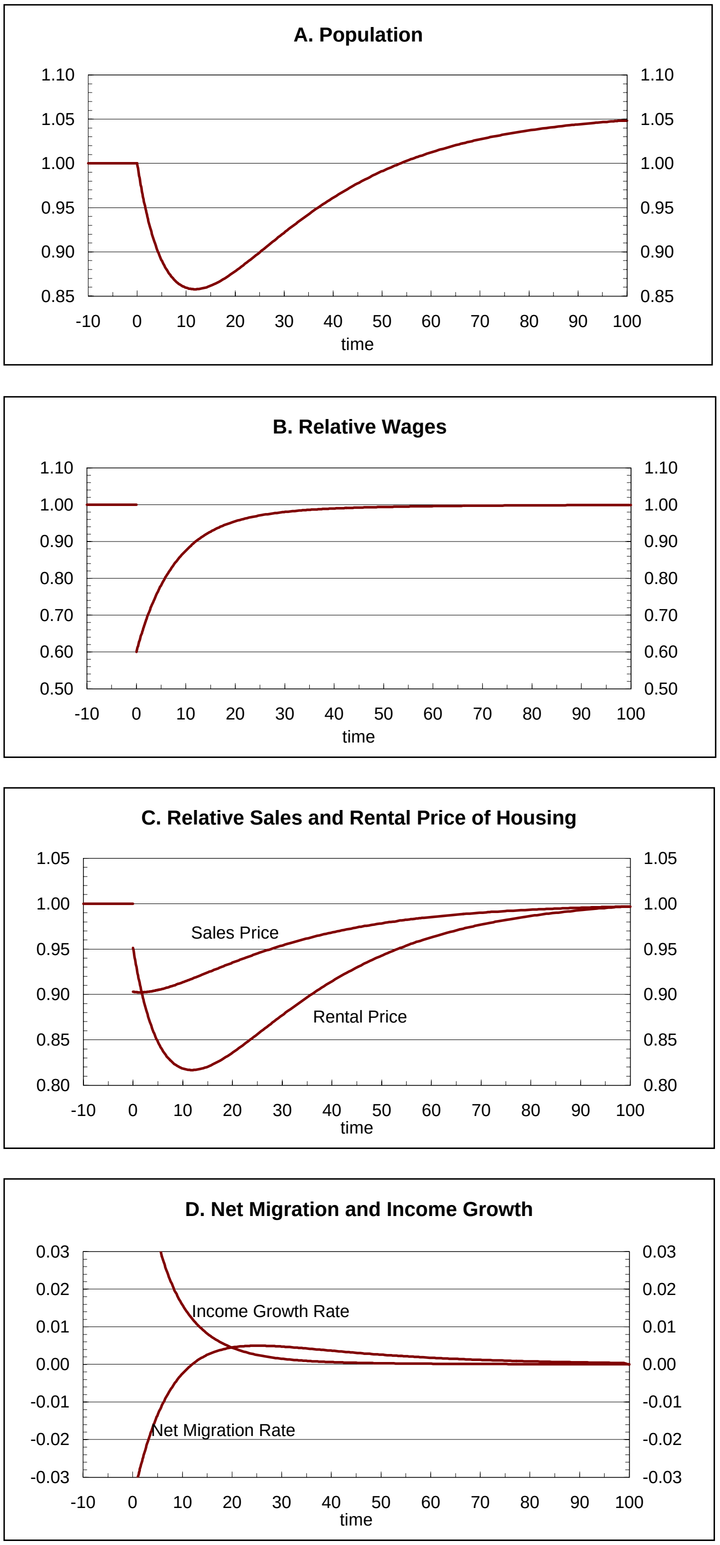
E. Base Capital Mobility (q_K* = 1.56), Broad Capital Share (α = 0.60)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
labor mobility	labor friction	Initial Growth Rates (at t= 0)			Relative Wage Minimum		House Value, Assets, Pop. (Initial, Steady State Levels)			Net Migration Rate at Time (in years)			Time to Close Dist. to S.S.	
	(ω)	capital	wages	house val	level	time	init. house	s.s. assts	s.s. pop	t=0	t=10	t=20	50%	75%
	1.000625	0.0061	-0.0136	0.0026	0.974	6.3	1.142	0.971	1.210	0.0288	0.0036	0.0023	9.6	29.2
high	1.00125	0.0057	-0.0094	0.0025	0.976	8.0	1.140	0.972	1.209	0.0213	0.0041	0.0024	10.6	30.1
	1.0025	0.0053	-0.0063	0.0025	0.979	9.9	1.136	0.974	1.209	0.0157	0.0046	0.0025	12.2	31.9
	1.005	0.0047	-0.0039	0.0024	0.983	12.2	1.131	0.978	1.208	0.0112	0.0048	0.0028	14.8	35.3
base	1.01	0.0040	-0.0025	0.0023	0.986	15.7	1.122	0.981	1.206	0.0082	0.0046	0.0029	18.8	41.7
	1.02	0.0032	-0.0015	0.0020	0.990	19.0	1.110	0.985	1.205	0.0057	0.0039	0.0028	25.1	52.9
	1.04	0.0024	-0.0008	0.0018	0.993	23.4	1.094	0.988	1.204	0.0038	0.0031	0.0024	35.1	72.0
low	1.08	0.0017	-0.0005	0.0014	0.995	28.3	1.076	0.992	1.203	0.0025	0.0022	0.0019	51.9	104.8
	1.16	0.0011	-0.0003	0.0011	0.997	33.3	1.056	0.995	1.202	0.0016	0.0015	0.0013	80.3	161.2
	1.32	0.0007	-0.0002	0.0008	0.998	39.1	1.039	0.996	1.201	0.0010	0.0009	0.0009	128.3	257.0

F. Low Capital Mobility (q_K* = 3.24), Broad Capital Share (α = 0.60)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
labor mobility	labor friction	Initial Growth Rates (at t= 0)			Relative Wage Minimum		House Value, Assets, Pop. (Initial, Steady State Levels)			Net Migration Rate at Time (in years)			Time to Close Dist. to S.S.	
	(ω)	capital	wages	house val	level	time	init. house	s.s. assts	s.s. pop	t=0	t=10	t=20	50%	75%
	1.000625	0.0039	-0.0145	0.0023	0.970	7.2	1.124	0.964	1.214	0.0280	0.0027	0.0018	14.1	46.7
high	1.00125	0.0037	-0.0099	0.0023	0.972	9.5	1.123	0.966	1.213	0.0203	0.0032	0.0019	14.9	47.3
	1.0025	0.0035	-0.0069	0.0023	0.974	12.1	1.120	0.968	1.212	0.0150	0.0038	0.0020	16.4	48.7
	1.005	0.0032	-0.0046	0.0022	0.977	15.3	1.116	0.971	1.211	0.0108	0.0042	0.0023	19.0	51.5
base	1.01	0.0028	-0.0030	0.0021	0.981	19.1	1.109	0.974	1.210	0.0077	0.0041	0.0025	23.2	56.9
	1.02	0.0024	-0.0018	0.0019	0.985	23.7	1.099	0.978	1.208	0.0053	0.0036	0.0025	29.7	67.2
	1.04	0.0019	-0.0011	0.0017	0.988	30.0	1.086	0.983	1.207	0.0037	0.0029	0.0022	40.1	85.6
low	1.08	0.0014	-0.0006	0.0014	0.992	37.1	1.070	0.987	1.205	0.0024	0.0021	0.0018	57.1	117.7
	1.16	0.0010	-0.0004	0.0011	0.994	45.0	1.053	0.991	1.203	0.0016	0.0014	0.0013	85.8	173.6
	1.32	0.0006	-0.0002	0.0008	0.996	53.9	1.037	0.994	1.202	0.0010	0.0009	0.0009	133.9	268.9

Figure 1: Time-Series Response to a Negative Capital Shock



Exogenous Parameters

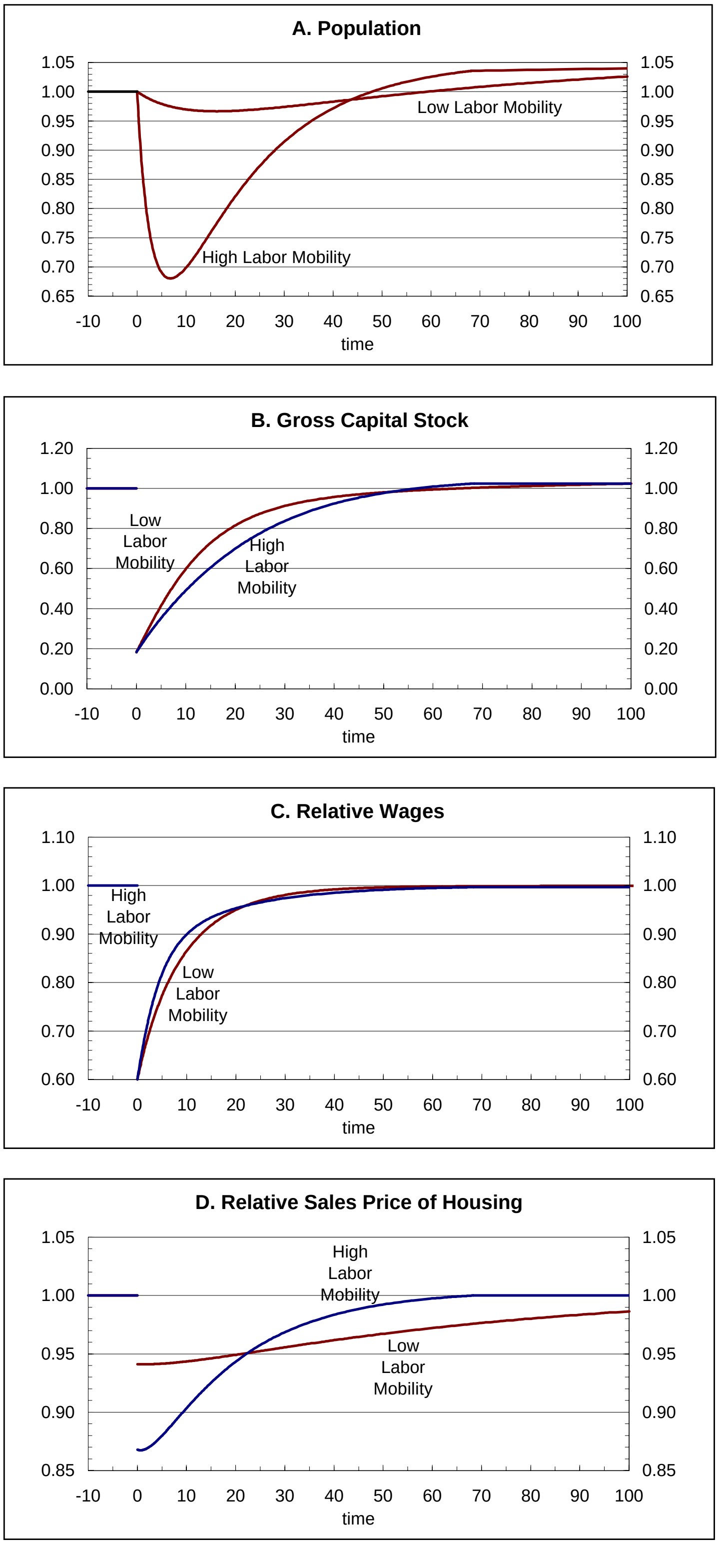
Figure assumes a shock which reduces initial physical capital stock such that income is at 60% of its steady-state level

Capital Share	α	= 0.30
Capital Depreciation Rate	δ	= 0.05
Housing Share	ζ	= 0.20
Time Preference	ρ	= 0.02
Technological Progress	x	= 0.02
Steady-State Shadow Value of Capital	q_K^*	= 1.56
Relative Wealth to Induce 1% Net Migration Rate	ω	= 1.01

Endogenous Results

Initial Relative House Sales Price	value =	0.903
Initial Net Migration	γ_L =	-0.035
Initial Rate Gross Capital Formation	γ_K =	0.240
Initial Income Growth	γ_w =	0.083
Minimum Population Density	L_{min} =	0.858
Steady-State Relative Population Density	L^* =	1.058
Steady-State Relative Asset Wealth	assts*=	0.447

Figure 2: High Versus Low Labor Mobility and Income Shocks



Exogenous Parameters

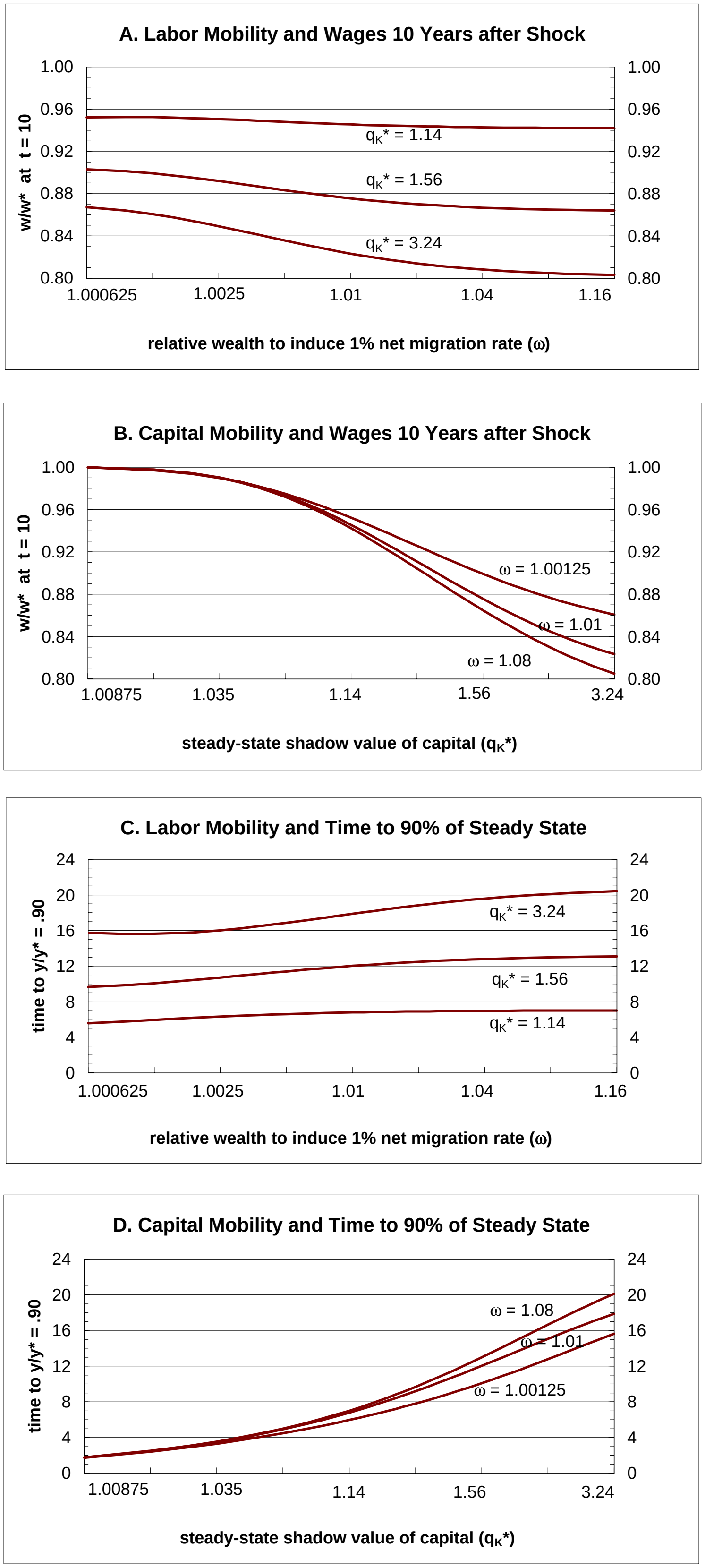
Figure assumes a shock which reduces initial physical capital stock such that income is at 60% of its steady-state level. Unless otherwise noted, parameters are the same as in Figure 1.

Relative Wealth to induce 1% Net Migration Rate	ω_{High} = 1.00125 ω_{Low} = 1.08000

Endogenous Results

Initial Relative House Sales Price	$Value_{High}$ = 0.863 $Value_{Low}$ = 0.936
Initial Net Migration	$\gamma_{L,High}$ = -0.159 $\gamma_{L,Low}$ = -0.006
Initial Gross Capital Formation	$\gamma_{K,High}$ = 0.204 $\gamma_{K,Low}$ = 0.259
Initial Income Growth	$\gamma_{w,High}$ = 0.109 $\gamma_{w,Low}$ = 0.080
Minimum Population Density	$L_{min,High}$ = 0.680 $L_{min,Low}$ = 0.966
Steady-State Relative Population Density	L^*_{High} = 1.053 L^*_{Low} = 1.060
Steady-State Relative Asset Wealth	$assts^*_{High}$ = 0.491 $assts^*_{Low}$ = 0.428

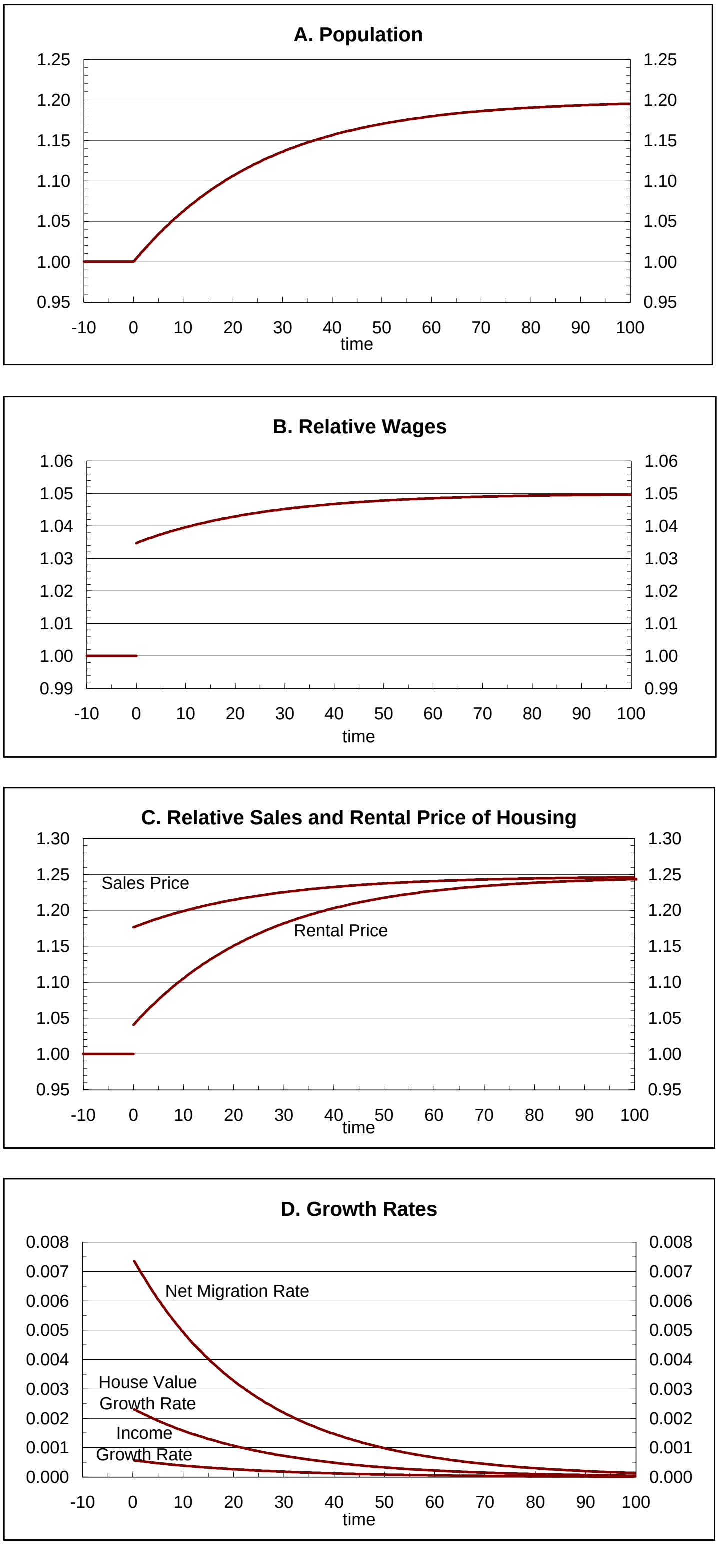
Figure 3: Mobility and Income Convergence



Exogenous Parameters

Figure assumes a shock which reduces initial physical capital stock such that income is at 60% of its steady-state level. Except as noted in figure, parameters are the same as in Figure 1.

Figure 4: Time-Series Response to a Positive Productivity Shock



Exogenous Parameters

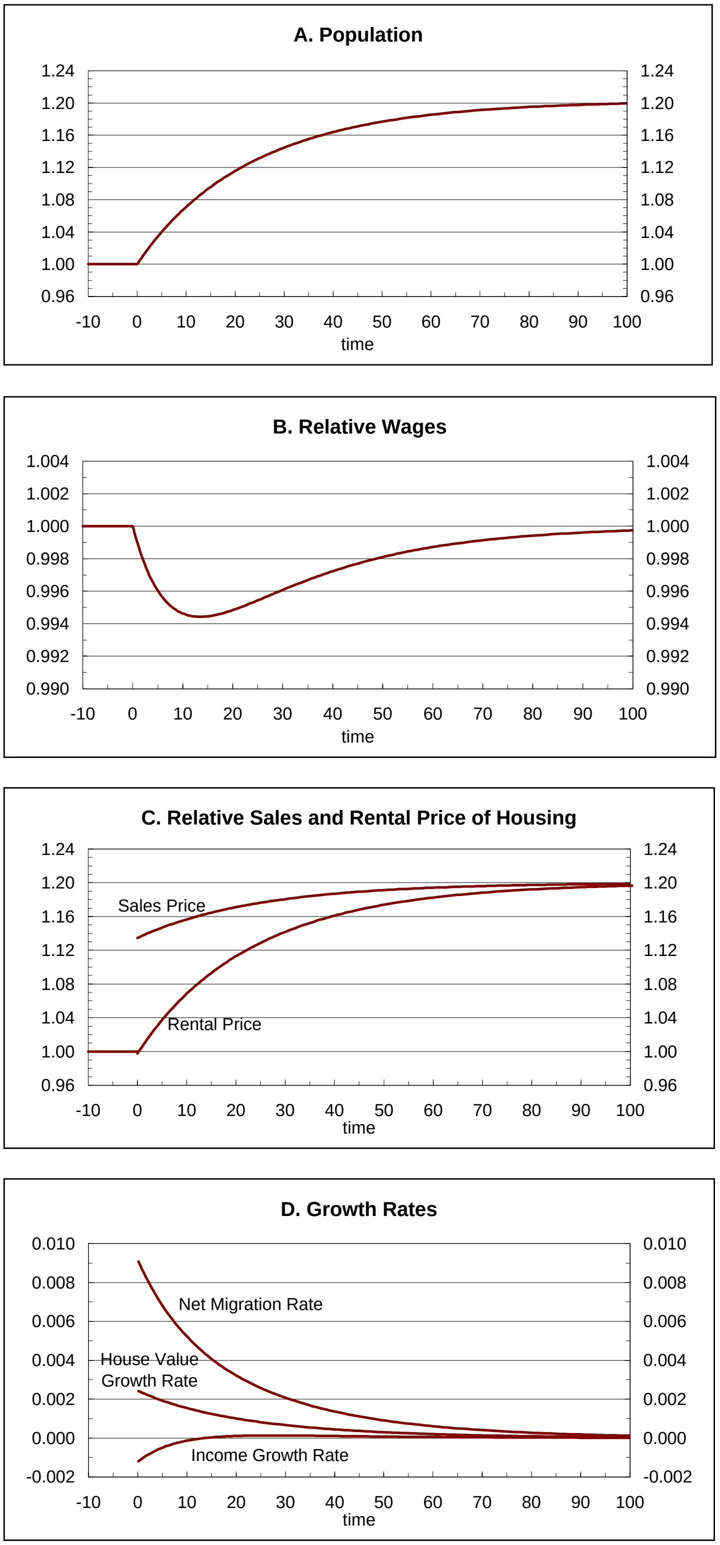
Figure assumes a shock which increases total factor productivity such that steady-state wages increase by 5%

Capital Share	α =	0.30
Capital Depreciation Rate	δ =	0.05
Housing Share	ζ =	0.20
Time Preference	ρ =	0.02
Technological Progress	x =	0.02
Steady-State Shadow Value of Capital	q_K* =	1.56
Relative Wealth to Induce 1% Net Migration Rate	ω =	1.01

Endogenous Results

Initial Relative House Sales Price	value =	1.176
Initial Net Migration	γ_L =	0.0073
Initial Rate Gross Capital Formation	γ_K =	0.0092
Initial Income Growth	γ_w =	0.0006
Initial House Sales Price Growth Rate	γ_v =	0.0023
Steady-State Relative Population Density	L* =	1.199
Steady-State Relative Asset Wealth	assts* =	0.953
Steady-State Relative House Sales Price	value* =	1.248

Figure 5: Time-Series Response to a Positive Quality-of-Life Shock



Exogenous Parameters

Figure assumes a shock which increases quality of life such that individuals are willing to pay 20% more for housing services while attaining reservation level of utility

Capital Share	α	= 0.30
Capital Depreciation Rate	δ	= 0.05
Housing Share	ζ	= 0.20
Time Preference	ρ	= 0.02
Technological Progress	x	= 0.02
Steady-State Shadow Value of Capital	$q_K{}^*$	= 1.56
Relative Wealth to Induce 1% Net Migration Rate	ω	= 1.01

Endogenous Results

Initial Relative House Sales Price	value =	1.135
Initial Net Migration	γ_L =	0.0089
Initial Rate Gross Capital Formation	γ_K =	0.0051
Initial Income Growth	γ_w =	-0.0011
Initial Land Sales Price Growth Rate	γ_v =	0.0024
Minimum Population Density	L_{min} =	0.994
Steady-State Relative Population Density	L^* =	1.203
Steady-State Relative Asset Wealth	assts*=	0.975